Going the Distance in Taxicab Land Assessment

Write a paragraph comparing and contrasting the characteristics of the geometric objects you studied in the Going the Distance in Taxicab Land lesson as they are represented on the traditional coordinate grid and as they are represented on a taxicab coordinate grid.

1. Be sure to include all of the following objects in your analysis:

   line segments
   circles
   perpendicular bisectors

2. Discuss whether you think the definitions or characteristics of these objects are valid and useful for both geometric systems.

3. Can you think of situations that might be better represented in Taxicab geometry than in traditional geometry?

Going the Distance in Taxicab Land Lesson:
Describing and Analyzing Objects in Two Different Geometric Systems.

Activity 1:

Draw accurate sketches to represent the following definitions, theorems, or postulates, and answer the questions that follow.

1. **Two points determine a line segment.**

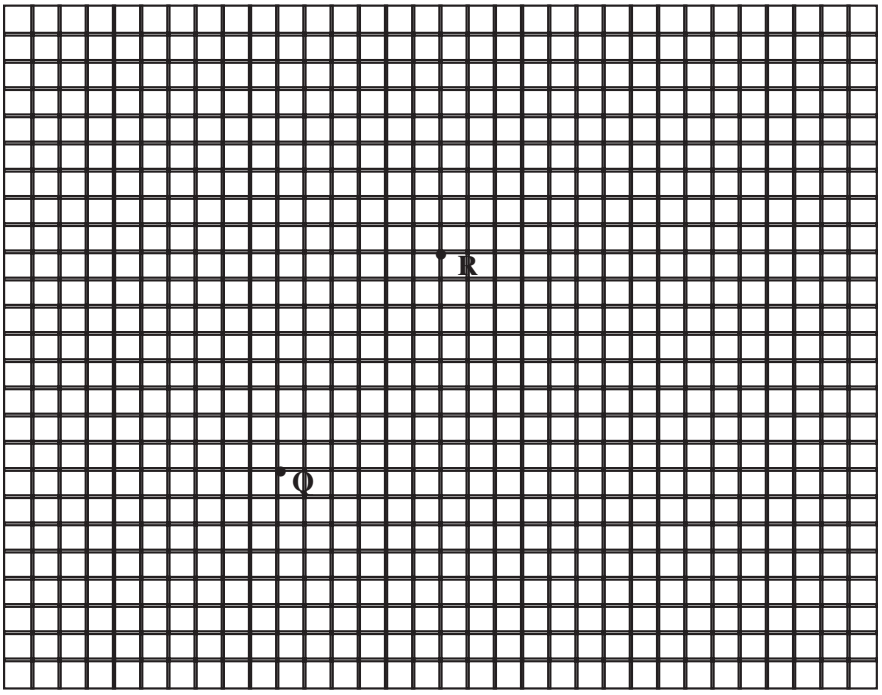
R

Q

a) How many line segments can be drawn between the two given points?

b) The line segment drawn between the two points represents the _____ distance between the two points.

2. Make the same sketch on a coordinate grid.



a) How many line segments can be drawn between the two points?

b) The line segment drawn between the two points represents the

_____ distance between the two points.

c) Does placing the geometric object on the coordinate grid change its characteristics? Explain your answer.

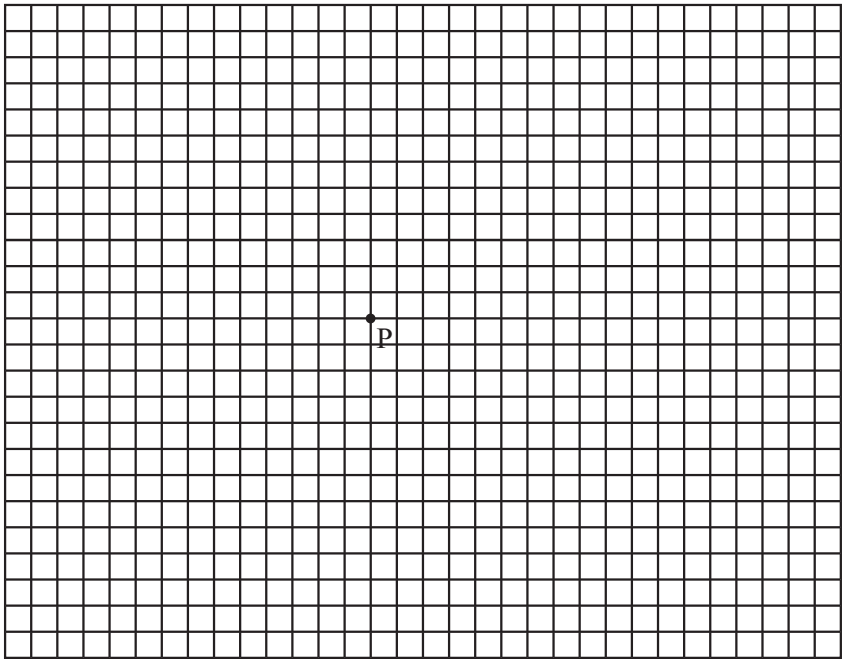
3. **A circle is the set of points in a plane that are the same distance from a given point in the plane.**

Sketch a circle with center at point P and a radius of 6 cm.

• P

a) How many different circles can you draw with this center and radius?

4. Make the same sketch on a coordinate grid. (For convenience, make the radius of the circle 6 grid units.)

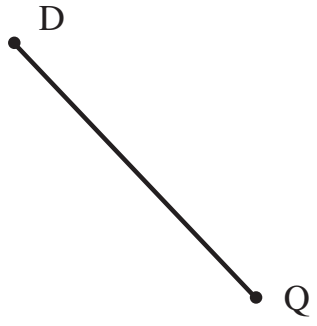


a) How many different circles can you draw with this center and radius?

b) Does placing the geometric object on the coordinate grid change its characteristics? Why?

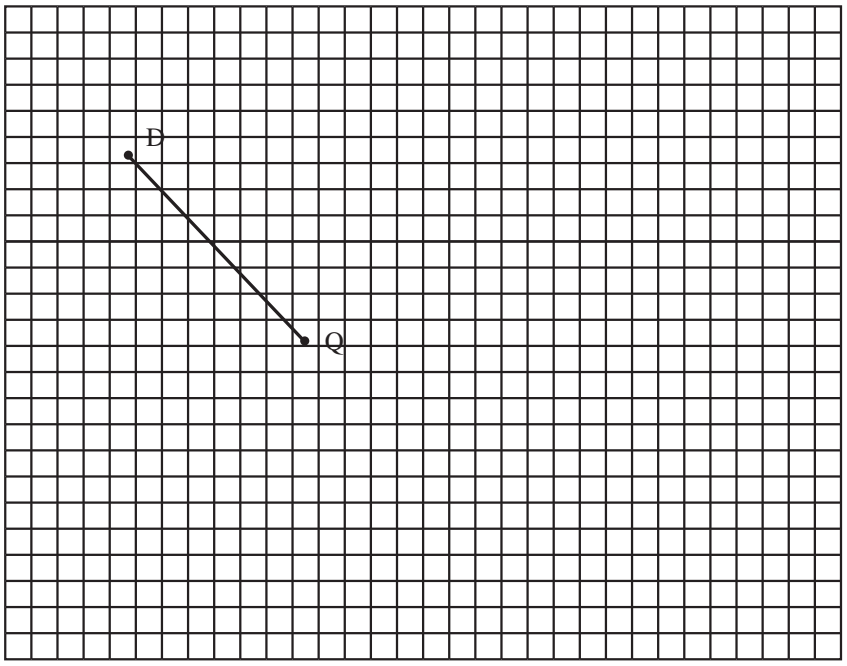
c) Does changing the location of the geometric object on the coordinate grid change its characteristics?

5. **A point is on a segment's perpendicular bisector if and only if it is the same distance from each of the segment's endpoints.** (Represent all the points that satisfy this requirement.)



a) How many different perpendicular bisectors is it possible to draw for the given line segment?

6. Make the same sketch on a coordinate grid.



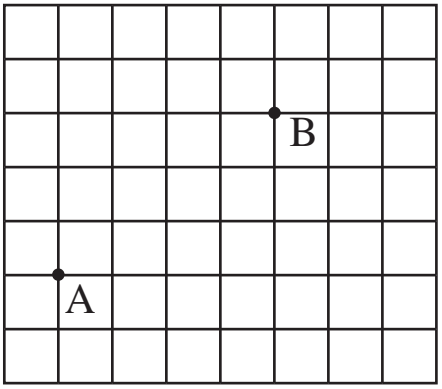
a) How many different perpendicular bisectors is it possible to draw for the given line segment?

b) Does placing the geometric object on the coordinate grid change its characteristics? Why or why not?

c) Describe how changing the position of the geometric object on the coordinate grid changes its characteristics.

Activity 2:

1. Determine the distance between points A and B on the coordinate grid below.



2. Imagine you are a taxicab driver and the coordinate grid above represents the grid of city streets you can travel on. Determine the **taxidistance** from point A to point B in Taxicab Land.

3. In what ways do you think the coordinate grid in Taxicab Land is different from the traditional coordinate grid you worked with in Activity 1?

Activity 3:

You have seen that representing a geometric object on a traditional coordinate grid does not change any of its characteristics. Neither does repositioning the object on the grid.

Your task is to analyze what happens when geometric objects are placed on a taxicab grid which has the following characteristics:

Points on a taxicab grid can only be located at the intersections of horizontal and vertical lines.

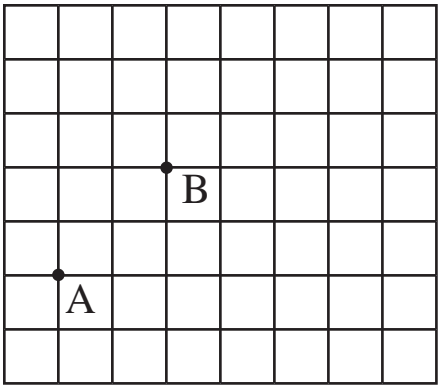
One unit will be one grid unit.

The numerical coordinates of points in **taxicab geometry** must therefore always be integers.

The **taxidistance** between 2 points is the smallest number of grid units that an imaginary taxi must travel to get from one point to the other. In Activity 2, the **taxidistance** between point A and point B is 7.

Draw accurate sketches on a **taxicab geometry** coordinate grid to represent the following definitions, theorems, or postulates, and answer the questions that follow.

1. **Two points determine a line segment.**

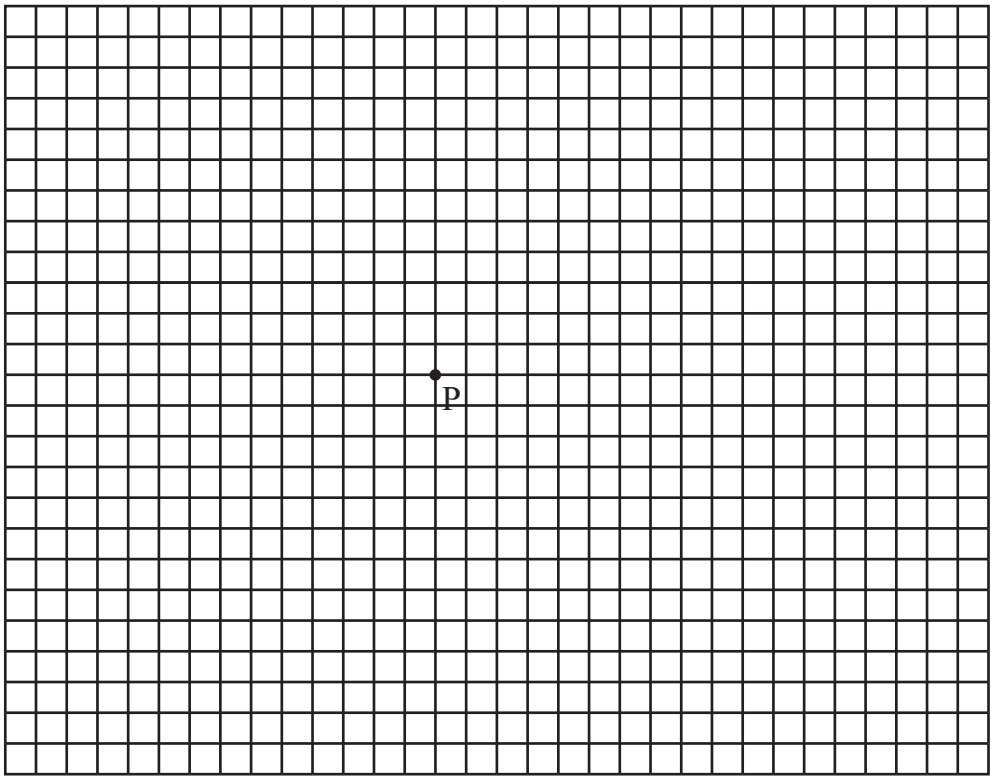


a) Is this the only line segment that can be drawn between the two given points? Explain.

b) The line segment(s) drawn between the two points represents the _____ taxidistance between the two points.

c) Does changing the position of the geometric object on the taxicab grid change its characteristics? Explain.

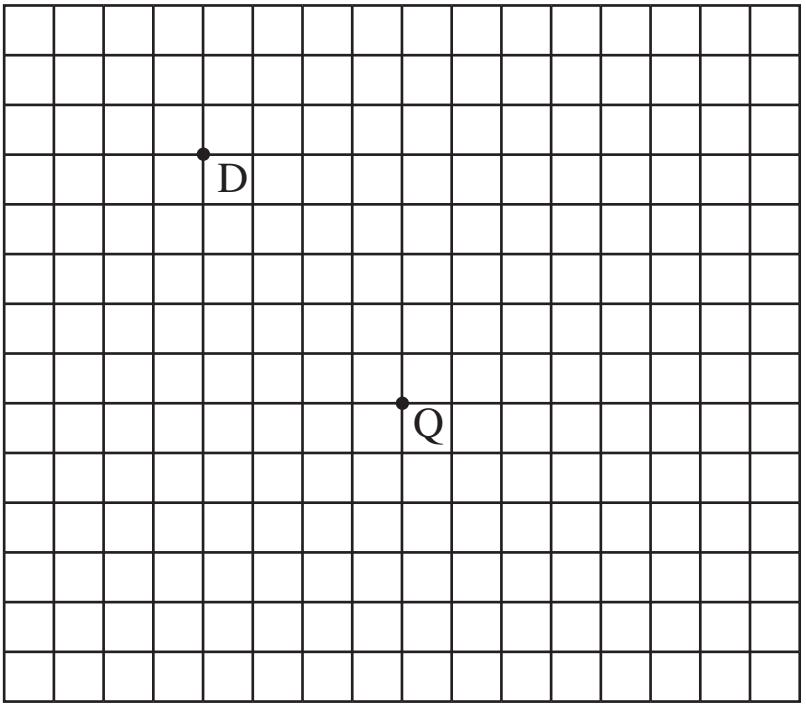
2. **A circle is the set of points in a plane that are the same distance from a given point in the plane.** (Sketch a circle with a radius of 6.)



a) How many different circles can you draw with this center and radius? Explain your answer.

b) Explain how changing the position of the geometric object on the taxicab grid changes its characteristics.

3. **A point is on a segment's perpendicular bisector if and only if it is the same distance from each of the segment's endpoints.** (Represent all the points that satisfy this requirement.)



a) Is the set of points you represented in your drawing the only perpendicular bisector for the two given endpoints? Explain.

b) Does changing the position of the geometric object on the coordinate grid change its characteristics? Explain.

Teacher Notes

Materials:
One compass and ruler per student.

Connections to Geometry TEKS:

(b.1) **Geometric structure.** The student understands the structure of, and relationships within, an axiomatic system.

The student:

(A) develops an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems;

(C) compares and contrasts the structures and implications of Euclidean and non-Euclidean geometries.

(d.2) **Dimensionality and the geometry of location.** The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.

The student:

(A) uses one- and two-dimensional coordinate systems to represent points, lines, line segments, and figures;

(C) develops and uses formulas including distance and midpoint.

(e.2) **Congruence and the geometry of size.** The student analyzes properties and describes relationships in geometric figures.

The student:

(A) based on explorations and using concrete models, formulates and tests conjectures

The lesson is intended to allow students to develop "an awareness of the structure of a mathematical system," (See TEK b.1 (A)). If students are already familiar with taxicab geometry, the assessment may be used to evaluate their understanding of this mathematical system.

Scaffolding Questions:

Assessment

Encourage students to review all the information in the lessons. They may want to organize the information in a chart form before they write their summary paragraph.

Activity 1:

1.

- What does the word "determine" mean in the sentence: "Two points determine a line segment"?

2.

- Is the coordinate grid line segment drawn between point Q and R unique? Does it represent the shortest distance between those two points?

4. Students may notice that point P does not lie at a grid intersection.
   - Is it possible to draw a circle with a radius of 6 with point P positioned this way?
   - After students draw the initial circle, they may position point P at a grid point and redraw the circle.

6. Students may notice that the segment does not have endpoints that lie on grid intersections.
   - Is it possible to draw the perpendicular bisector with the points in their present position?
   - Students may reposition the segment so that its endpoints lie on grid intersections and then complete the sketch.

Activity 2:

1.

- Is it possible to find the distance between A and B by counting along the grid lines?
- What formula could we use to determine the distance between A and B?

2.

- In Taxicab Land is it possible to find the distance between A and B by counting along the grid lines?
- Is there more than one way to count out the distance?
- Which way (or ways) of counting out the distance do you think is correct?

3.

- Where can points be located on a traditional coordinate grid?
- Where can points be located on a taxicab coordinate grid?
- What does this tell us about the numerical coordinates of the points on each type of coordinate grid?
- How is the distance between two points on a taxicab coordinate grid different from the distance on a traditional coordinate grid?

Activity 3:

1. Students may notice that there are many ways to draw a minimum distance pathway between the two points.
   - Do all these segments represent the shortest taxidistance between the two points?

2.

- Do the points you drew satisfy the definition of a circle?
- Should you connect the points by drawing along the grid lines so that it looks more like an enclosed shape?

about the properties of parallel and perpendicular lines;

(C) based on explorations and using concrete models, formulates and tests conjectures about the properties and attributes of circles and the lines that intersect them.

Texas Assessment of Knowledge and Skills:

Objective 7. The student will demonstrate an understanding of two and three dimensional representations of geometric relationships and shapes.

Connection to High School Geometry: Supporting the TEKS and TAKS Institute:

I. Structure: Bayou City Geometry

Student's Response to Assessment Question 2:

"In normal Geometry, a line is the shortest distance between two points. In Taxicab it is not. In normal Geometry circles have points in which the shortest distance from the center to each point is equal. In taxicab geometry, the circles are different. They are not curved. In normal geometry, all segments have a perpendicular bisector. In taxicab geometry only segments with an even taxicab distance have perpendicular bisectors. The definitions for geometric shapes are only useful in normal geometry, not taxicab. You would use taxicab geometry if you wanted to find out how far a car travel to get from one street corner to another."

3.

- Do you think there will be any points that are the same distance from both endpoints?

Sample Solution:

Assessment

Traditional Coordinate System

Line Segment:	Uniquely determined by two points. Represents shortest distance between two points.
Circle:	Uniquely determined by given center and radius. Characteristics do not change when repositioned.
Perpendicular Bisector:	Uniquely determined by endpoints of segment. Characteristics do not change when repositioned.

Taxicab Coordinate System

Line segment:	Not necessarily uniquely determined by two points. Represents shortest distance between two points.
Circle:	Uniquely determined by center and radius. Characteristics do not change when repositioned.
Perpendicular Bisector:	May not exist at all. If it does exist, however, then it is uniquely determined by endpoints of segment. Characteristics change when repositioned.

Students should realize that the definitions and characteristics may be more useful and familiar in the traditional system, but they are equally valid in both systems.

Any sort of problem where one needs to find locations along a grid of city streets might be more successfully represented using taxicab geometry.

Activity 1:

1. a) One

   b) The line segment drawn between the two points represents the shortest distance between the two points.

2. a) One

   b) The line segment drawn between the two points represents the shortest distance between the two points.

   c) No. There is one and only one line segment between the two points, and it represents the shortest distance between them.

3. Draw circle with a compass.

   a) Only one circle may be drawn with a given radius.

4. Circle can be drawn on coordinate grid with a radius of 6 grid units by using a compass.

   If students subsequently reposition point P on a grid point, then they can sketch the circle by counting grid units, make a ruler in grid units, etc.

   a) One

   b) No. There is only one circle with the given radius and center.

   c) No. The coordinate name of the point changes but the properties of the circle stay the same.

5. Compass and straightedge construction is the most accurate way to do this.

   a) One

6. Compass and straightedge construction is still an appropriate way to do this. If students reposition points D and Q to the nearest grid intersections, then the 10 unit segment's midpoint will also fall on a grid intersection.

   a) One

   b) No. There is only one set of points that satisfy the requirement of being the same distance from the endpoints of segment $\overline{DQ}$.

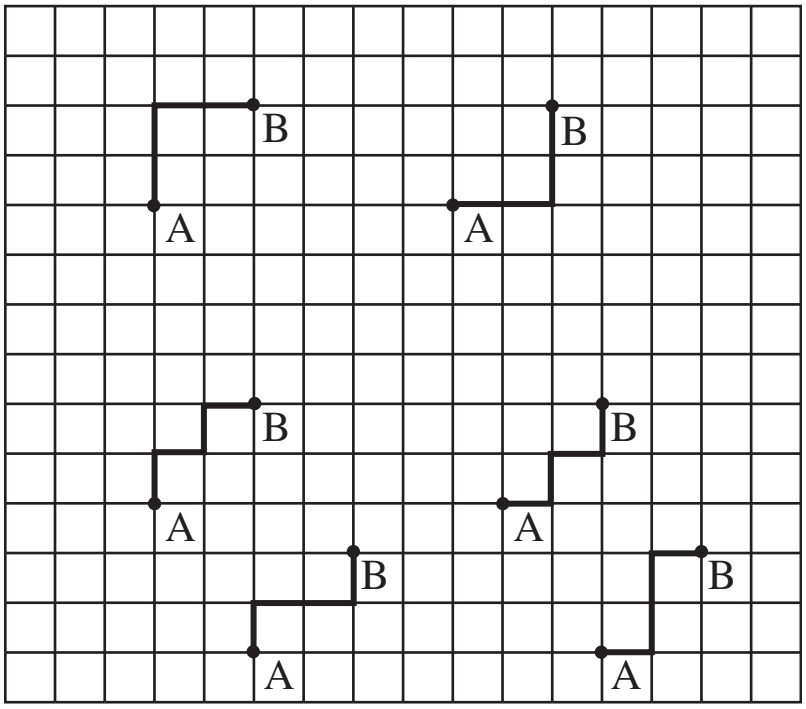
   c) No

Activity 2:

1. $\overline{AB}$ is the hypoteneuse of a right triangle with sides of length 3 units and 4 units. By the Pythagorean Theorem the length of $\overline{AB}$ is 5 units.
2. The taxi driver must travel on the streets so he would travel 3 blocks vertically and 4 blocks horizontally. He would travel a total distance of 7 blocks.
3. Points on a taxicab grid can only be located at grid intersections. This makes their numerical coordinates integers. The taxidistance between two points must always be an integer.

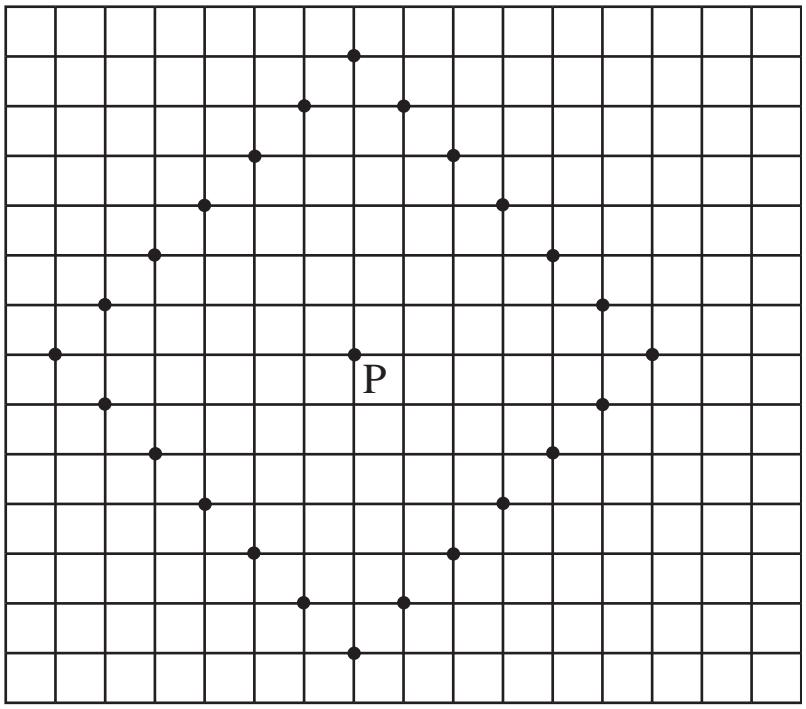
Activity 3:

1. a) There are six minimum distance pathways that can be drawn between the two points.



b) The line segment(s) drawn between the two points represent the <u>shortest</u> taxidistance between the two points.

c) Yes. Students should realize that if they position the two points along either a horizontal or a vertical grid line, then there will only be one minimum distance segment between them.

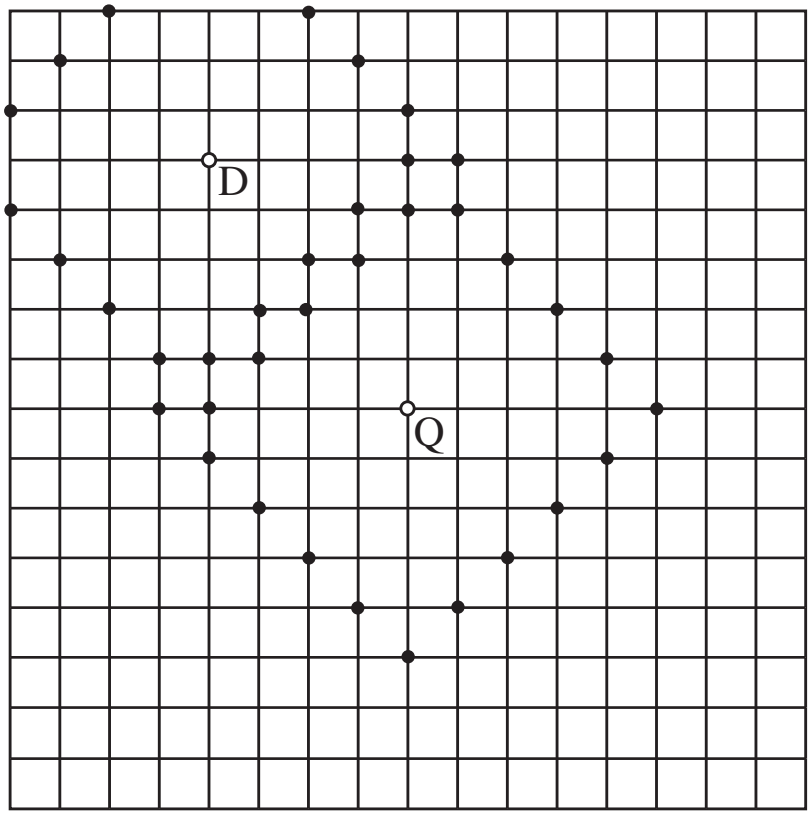


2. a) There is only one set of points that are all 6 units from P. They are points that lie on "street corners," such that the sum of the horizontal and vertical distances is 6 units.

   b) The coordinates of the points would change, but the shape of the figure would remain the same.

3. a) There are no points that satisfy the requirement of being the same distance from points D and Q. The distance from point D to point Q is 9 units.

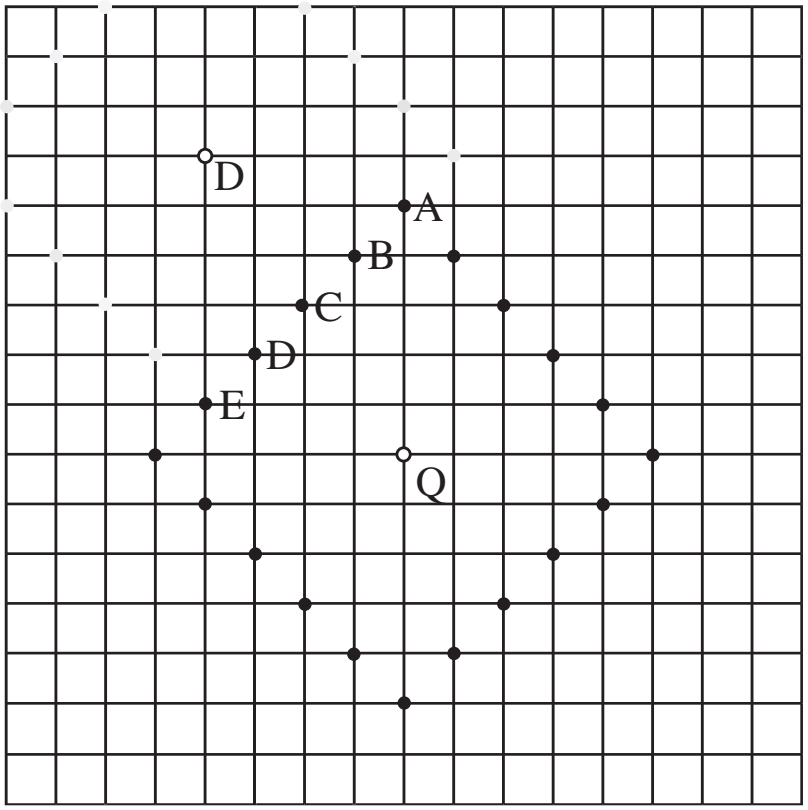
   The graph shows the set of points that are 5 units from D and the set of shaded points that are 5 units from Q. There are no common points. The two taxicab circles will not intersect.



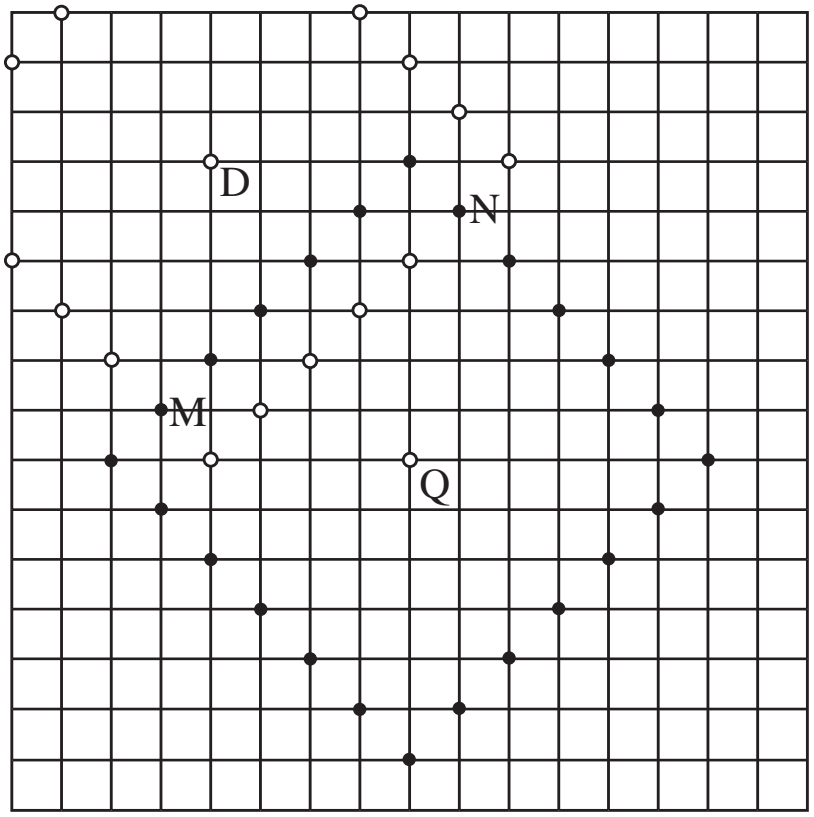
b) Students may experiment and discover that if the taxidistance between the two points is odd, then there will be no perpendicular bisector.

If, however, the taxidistance between the two points is even, there will be a perpendicular bisector, and it can take on a variety of configurations, depending on how the points are positioned.

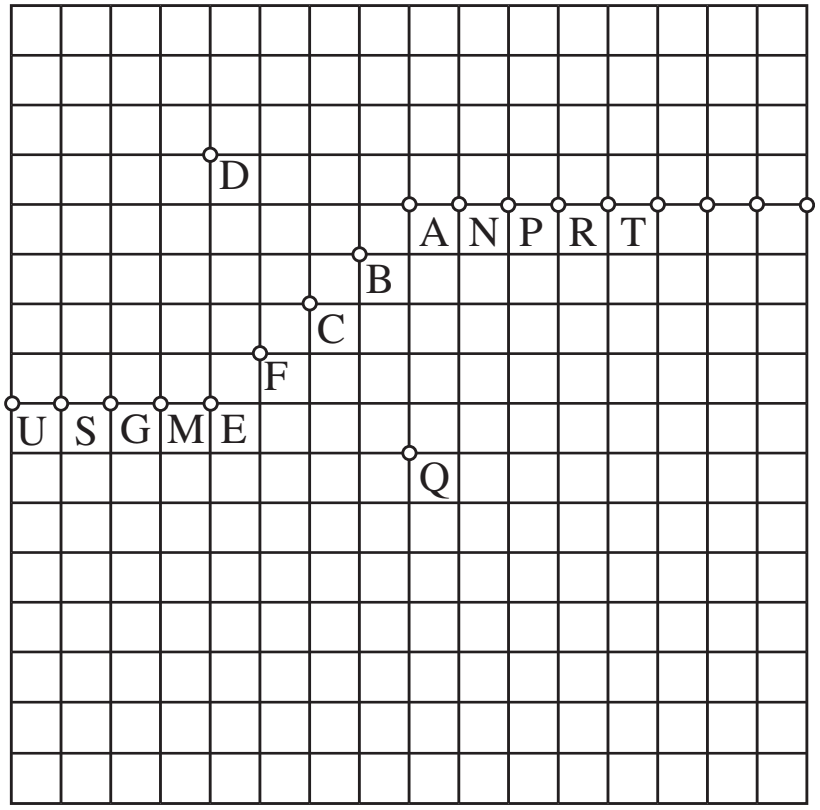
D and Q in this graph are 10 units apart. The next graph shows the two circles that have a radius of 5 units and centers D and Q. The points that the two circles have in common (A, B, C, D, and E) are all 5 units from both D and Q. Then A, B, C, D, and E are points on the perpendicular bisector of D and Q.



The next graph shows the two circles that have a radius of 6 units and centers D and Q. The points that the two circles have in common, M and N, are 6 units from both D and Q.



This next graph shows the points A, B, C, D, and E , 5 units from D and Q, M and N (six units from D and Q), P and Q (seven units from D and Q), R and S (eight units from D and Q), T and U (nine units from D and Q). This collection of points is a part of the set of points equidistant from D and Q.



Extension Questions:

- For each pair of points given, decide if it is possible to draw a perpendicular bisector in a taxicab coordinate system.

(1, 3) and (5, 3)	*yes*
(-2, 0) and (-6, -4)	*yes*
(5, -2) and (2, 1)	*yes*
(-3, 3) and (2, 1)	*no*
(5, -2) and (-3, 3)	*no*
(5, 3) and (5, -1)	*yes*

- Write a conjecture about what must be true in order for it to be possible to draw a perpendicular bisector in a taxicab coordinate system.

  The taxidistance between the two endpoints must be an even number.

- Experiment with circles of varying radii on a taxicab coordinate system. Write a conjecture about the value of π in taxicab geometry.

  The value of π in taxicab geometry is 4.