**Lesson Description: Pick's Theorem Investigation**

**Overview:**  The purpose of this activity is to lead students through a series of activities which ultimately result in the "discovery" of Pick's Theorem. The lesson begins with a "real-life" problem which requires finding the area of a lattice polygon. Students can do this either by counting and estimating square units or by dividing the figure up into known polygons. In order to explore whether or not there is an algebraic formula to help find such an area, students are first lead to consider lattice polygons with no interior pegs. Students find areas for such polygons with three, four, five, and six boundary pegs and determine the corresponding areas. By building a table, the student can look for a pattern and determine a formula for the area (A) of a lattice polygon with no boundary pegs and *i* interior pegs. This work continues for lattice polygons with one interior peg, two interior pegs, etc., and again by patterning, the student can be lead to write Pick's Formula - the formula for computing the area (A) of any lattice polygon with *i* interior pegs and *b* boundary pegs. Follow-up activities with students include application problems for this formula. Follow-up activities for teachers include a discussion of the positive characteristics of this problem/lesson. Extension activities and provided and discussed.

**Materials:** geoboards, bands, several handouts, pencils, calculators

**Ohio's Learning Standards**

**Grade 6 - Critical Areas 3 and 5**

**6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers.

**6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

**6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

**6.G.1** Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.

**Grade 7 - Critical Areas 2 and 3**

**7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

**7.G.6** Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and

right prisms.

**Grade 8 - Critical Areas 1 and 2**

**8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

**Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

**NCTM Process Standards (2000):** Problem solving, Reasoning and Proof, Communication, Connections, Representations

**Problem/Lesson Characteristics for Teachers to Consider**

content motivated by an initial problem

"real life" problem to be solved

multiple tools - geoboard, grid paper, formula, calculator

illustrates movement from concrete to pictorial to abstract

illustrates the use of a graphic organizer

uses multiple and varied problem-solving strategies

includes student conjecturing/testing

allows students to "create" mathematics

gives students a sense of mathematical power

illustrates an inductive approach

connects to history

illustrates the teacher as facilitator

has potential for differentiated instruction

provides opportunities for student reasoning/mathematical thinking

integrates mathematics topics (geometry, algebra, measurement, functions)

illustrates Standards for Mathematical Practice