**Areas of Lattice Polygons**

The polygon which represents Nate and Ashley's ceiling is referred to as a *lattice polygon* as it appears on a lattice or grid paper. Let's investigate a process for finding areas of lattice polygons. Use the geoboards for your explorations.

Nate and Ashley's ceiling diagram had *lattice points* on the border and *lattice points* in the interior. For the time being, let's just explore lattice polygons with no interior points.

What is the minimum number of boundary points required for creating a lattice polygon?

Make a table which shows the relationship between the number of boundary points (b) and the area (A) of the lattice polygon **assuming there are no interior points.**  Again, use your geoboard to explore.

**Table 1: Areas of Lattice Polygons with No Interior Points**

|  |  |
| --- | --- |
| **Number of Boundary Points (b)** | **Area of Lattice Polygon (A)** |
| **3** |  |
| **4** |  |
| **5** |  |
| **6** |  |
| **7** |  |

Describe this relationship in words.

Write a formula which represents this relationship.

Now consider lattice polygons with exactly one interior peg. Create a few on your geoboard. Make a table which shows the relationship between the number of boundary points (b) and the area (A) of the lattice polygon assuming that there is exactly one interior peg.

**Table 2: Areas of Lattice Polygons with One Interior Point**

|  |  |
| --- | --- |
| **Number of Boundary Points (b)** | **Area of Lattice Polygon (A)** |
| **3** |  |
| **4** |  |
| **5** |  |
| **6** |  |
| **7** |  |

Describe this relationship in words.

Write a formula to represent this relationship.

Transfer the information from Tables 1 and 2 onto Table 3 on the next page.

**Table 3: Areas of Lattice Polygons**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Number of Boundary Pegs (b)** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **b** |
| **Number of Interior Pegs (i)** | **XXXX** | **XXXX** | **XXXX** | **XXXX** | **XXXX** | **XXXX** | **XXXX** | **XXXX** | **XXXX** |
| **0** | **XXXX** |  |  |  |  |  |  |  |  |
| **1** | **XXXX** |  |  |  |  |  |  |  |  |
| **2** | **XXXX** |  |  |  |  |  |  |  |  |
| **3** | **XXXX** |  |  |  |  |  |  |  |  |
| **4** | **XXXX** |  |  |  |  |  |  |  |  |
| **5** | **XXXX** |  |  |  |  |  |  |  |  |
| **i** | **XXXX** |  |  |  |  |  |  |  |  |

Repeat the process above for exactly two interior pegs and for exactly three interior pegs. You may use your geoboard or grid paper for your explorations. Fill in the table for those two rows above.

Write a formula for the area (A) of a lattice polygon with two interior pegs and b boundary pegs.

Write a formula for the area (A) of a lattice polygon with three interior pegs and b boundary pegs.

Copy below the formulas you've created for the area (A) of a lattice polygon with 0, 1, 2, or 3 interior pegs and b boundary pegs.

Look for patterns in these formulas and create a formula for the area (A) of any lattice polygon with i interior pegs and b boundary pegs.

Use your formula to find the areas of the lattice polygons on the page provided.

Use other methods to verify these answers.

Use your formula to verify the area of Nate and Ashley's ceiling.