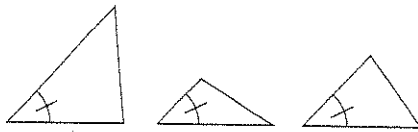


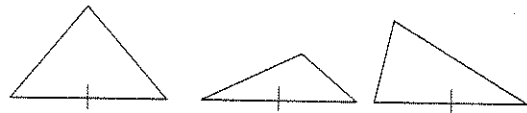
## CONGRUENT TRIANGLES

In the patty paper geometry investigations that follow you are going to investigate conditions that make triangles congruent. Do you have to know that all six parts (the three angles and the three sides) of one triangle are congruent to all six corresponding parts of another triangle before you know that the two triangles are congruent? Or, can you conclude that two triangles are congruent knowing that fewer than six pairs of parts are congruent?

It is obvious that if you only know one side or one angle you don't have enough information to form a unique triangle.



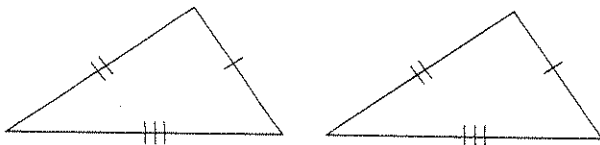
**one angle**



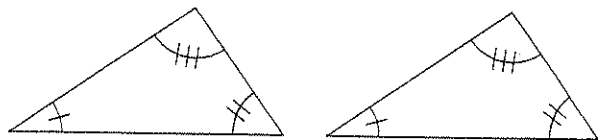
**one side**

What if you know the lengths of two sides, the measures of two angles, or the length of a side and the measure of an angle? Convince yourself whether any of these situations provide you with enough information to draw a unique triangle.

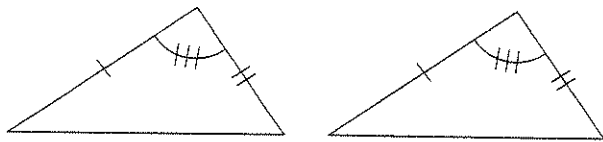
Will *three* parts of a triangle give you enough information to construct a unique triangle? That is the question you will be exploring in these investigations. There are six situations you will need to investigate. They are shown below.



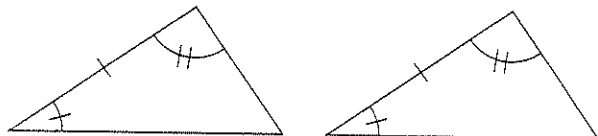
**Side-Side-Side**



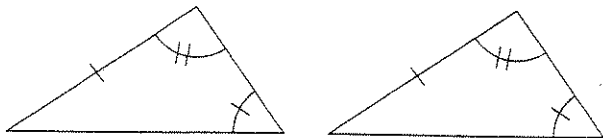
**Angle-Angle-Angle**



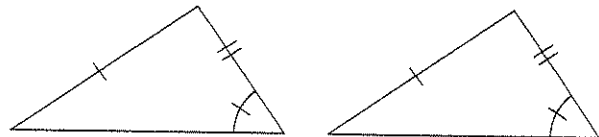
**Side-Angle-Side**



**Angle-Side-Angle**



**Side-Angle-Angle**



**Side-Side-Angle**

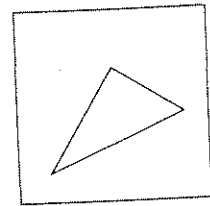


# Open Investigation 8.1

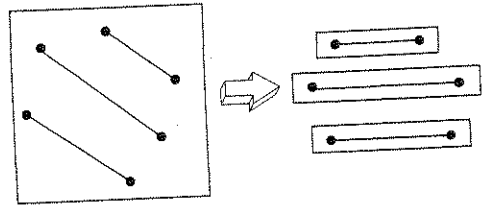
## SIDE-SIDE-SIDE

If the three sides of one triangle are congruent to the three sides of another triangle (SSS), then are the two triangles congruent?

**Step 1:** Draw a large scalene triangle on your patty paper.



**Step 2:** Copy the three sides separately onto another patty paper, and mark a dot at each endpoint. Cut the patty paper into three strips with one side on each strip.



**Step 3:** Arrange the three segments into a triangle by placing one endpoint on top of another.

**Step 4:** With a third patty paper trace the triangle formed. Compare the new triangle with the original triangle. Are they congruent? (You may have to flip over the patty paper with the traced triangle.)

**Step 5:** Try rearranging the three segments into another triangle. Can you create a triangle that is not congruent to the original triangle?

Compare your results with the results of others near you. If the triangles you formed with the three segments were always congruent to your original triangle, then you have demonstrated the SSS congruence shortcut for showing that two triangles are congruent.

If not, then you have found a counterexample, in which case SSS is not a shortcut for showing that two triangles are congruent.

Write a conjecture about two triangles if you know that three sides of one triangle are congruent to three sides of the other triangle.



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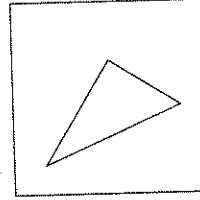


## Open Investigation 8.2

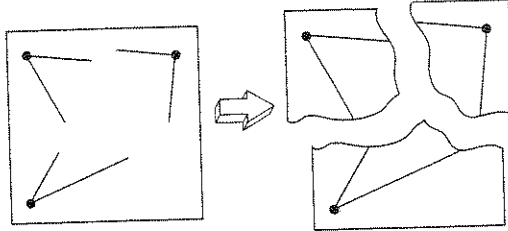
### ANGLE-ANGLE-ANGLE

If the three angles of one triangle are congruent to the three angles of another triangle (AAA), then are the two triangles congruent?

**Step 1:** Construct a large scalene triangle on your patty paper.



**Step 2:** Mark a dot in three of the four corners of another patty paper. Copy the three angles of your original triangle separately onto this second patty paper using each dot as a vertex of one of the angles. Cut the patty paper into three sections with one angle on each section. Extend each ray to the edge of the patty paper section.



**Step 3:** Arrange the three angles into a triangle by overlapping pairs of rays.

**Step 4:** On a third patty paper draw the triangle formed.

Compare the new triangle with the original triangle. Are they congruent? Compare your results with the results of others near you.

If the triangles formed with the three angles were always congruent to the original triangle, then AAA is a shortcut for showing that two triangles are congruent.

If not, you have found a counterexample which shows that AAA is not a shortcut for showing that two triangles are congruent.

Write a conjecture about two triangles if you know that three angles of one triangle are congruent to three angles of the other triangle.



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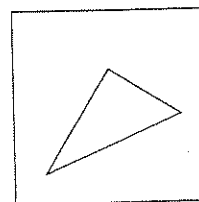


## Open Investigation 8.3

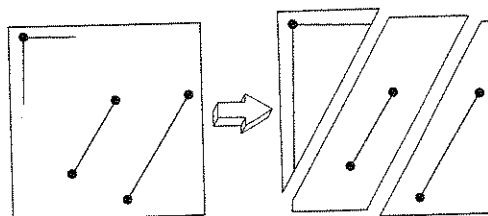
### SIDE-ANGLE-SIDE

If two sides and the angle between them of one triangle are congruent to two sides and the angle between them of another triangle (SAS), then are the two triangles congruent?

**Step 1:** Construct a large scalene triangle on your patty paper.



**Step 2:** Copy any two sides and the angle between them separately onto a second patty paper as before. Mark a dot on the vertex of the angle and on each endpoint of each side. Cut the patty paper into three sections with each part on a different section. Extend each ray of the copied angle to the edge of its patty paper section.



**Step 3:** Arrange the three parts into a triangle.

**Step 4:** Place a third patty paper over the three parts, mark a dot at each endpoint, and draw the triangle formed.

Compare the new triangle with the original triangle. Are they congruent? Can you create a triangle that is not congruent to the original triangle? Compare your results with the results of others near you.

If the triangles you formed with the same two sides and the same angle between them were always congruent to your original triangle, then you've demonstrated the SAS shortcut for showing that two triangles are congruent.

If not, then you have found a counterexample, and thus SAS is not a shortcut for showing that two triangles are congruent.

Write a conjecture about two triangles if you know that two sides and the angle between them of one triangle are congruent to two sides and the angle between them of the other triangle.



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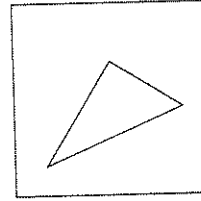


## Open Investigation 8.4

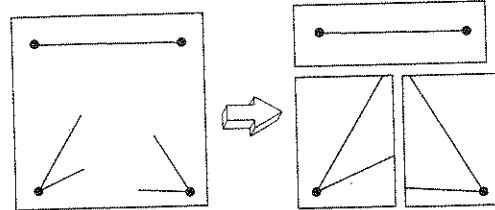
### ANGLE-SIDE-ANGLE

If two angles and the side between them of one triangle are congruent to two angles and the side between them of another triangle (ASA), then are the two triangles congruent?

**Step 1:** Construct a large scalene triangle on your patty paper.



**Step 2:** Copy any two angles and the side between them of your original triangle separately onto a second patty paper as before. Mark a dot on each endpoint of the copied segment and the vertex of each angle. Cut the patty paper into three sections with each part on a different section. Extend each ray of each copied angle to the edge of the patty paper section.



**Step 3:** Arrange the three parts into a triangle, making sure that you place the side between the two angles.

**Step 4:** Place a third patty paper over the three parts, mark a dot at each endpoint or vertex, and complete the triangle.

Compare the new triangle with the original triangle. Are they congruent? Can you create a triangle that is not congruent to the original triangle? Compare your results with the results of others near you.

If the triangles you formed with two angles and the side between them were always congruent to your original triangle, then you've demonstrated that ASA is a congruence shortcut.

If not, then ASA is not a shortcut for showing that two triangles are congruent.

Write a conjecture about two triangles if you know that two angles and the side between them of one triangle are congruent to two angles and the side between them of the other triangle.



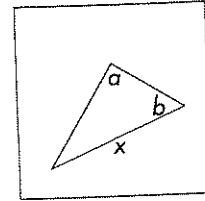


## Open Investigation 8.5

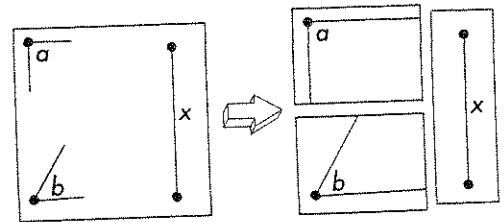
### SIDE-ANGLE-ANGLE

If two angles and a side not between them of one triangle are congruent to two angles and the corresponding side not between them of another triangle (SAA), then are the two triangles congruent?

- Step 1:** Draw a triangle on your patty paper. Label any two angles and a side not between them as shown.



- Step 2:** Copy the two angles and the side separately onto a second patty paper. Mark a dot on the vertex of each angle and on each endpoint of the side. Cut the patty paper into three sections with each part on a different section. Extend each ray of each copied angle to the edge of its patty paper section.



- Step 3:** Arrange the three parts into a triangle, being sure to place them in the same sequence as in your original triangle.

- Step 4:** Place a third patty paper over the three parts, make a dot at each endpoint or vertex, and draw the triangle formed.

Compare the new triangle with the original triangle. Are they congruent? Can you create a triangle that is not congruent to the original triangle? Compare your results with the results of others near you.

If the triangles you formed with the two angles and the side not between them were always congruent to your original triangle, then you've demonstrated that SAA is a shortcut for showing that two triangles are congruent.

If not, then you have found a counterexample, and SAA is not a shortcut to showing that two triangles are congruent.

Write a conjecture about two triangles if you know that two angles and a side that is not between them of one triangle are congruent to two angles and the corresponding side of the other triangle.

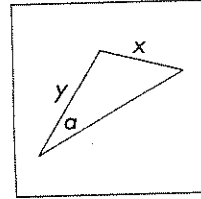


## Open Investigation 8.6

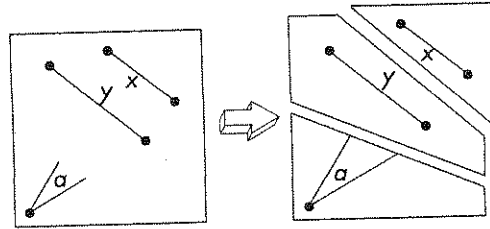
### SIDE-SIDE-ANGLE

If two sides and an angle not between them of one triangle are congruent to two sides and the corresponding angle not between them of another triangle (SSA), then are the two triangles congruent?

**Step 1:** Construct a large acute scalene triangle on your patty paper. Label any two sides and an angle not between them as shown.



**Step 2:** Copy the two sides and the angle separately onto a second patty paper. Mark a dot on the vertex of the angle and on each endpoint of each side. Cut the patty paper into three sections with each part on a different section. Extend each ray of the copied angle to the edge of the patty paper section.



**Step 3:** Arrange the three parts into a triangle, making sure the parts are in the same sequence as in your original triangle.

**Step 4:** Place a third patty paper over the three parts, mark a dot at each endpoint or vertex, and draw the triangle formed.

Compare the new triangle with the original triangle. Are they congruent? Can you create a triangle that is not congruent to the original triangle? Compare your results with the results of others near you.

If the triangles you formed with the two sides and the angle not between them were always congruent to your original triangle, then you've demonstrated that SSA is a congruence shortcut.

If not, then you have found a counterexample, and SSA is not a shortcut for showing that two triangles are congruent.

Write a conjecture about two triangles if you know that two sides and an angle not between them of one triangle are congruent to two sides and the corresponding angle of the other triangle.



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