Extending Children's Mathematics

Fractions and Decimals

SUSAN B. EMPSON

LINDA LEVI

Foreword by Thomas P. Carpenter

HEINEMANN

Portsmouth, NH

Heinemann
361 Hanover Street
Portsmouth, NH 03801–3912
www.heinemann.com

Offices and agents throughout the world

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Excerpt adapted from "Equal Sharing and Shared Meaning: The Development of Fraction Concepts in a First-Grade Classroom" by Susan B. Empson from *Cognition and Instruction*, January 9, 1999, Volume 17, Issue 3, pages 283–342. Published by Taylor and Francis Group, http://www.informaworld.com. Reprinted by permission of the publisher.

Introduction, Chapter 4, and Figure 7–1 photos courtesy of Lynne Nielsen; Chapter 1, Chapter 2, Figure 4–6, Figure 5–5, Chapter 8, Figure 8–1, and Figure 9–1 photos courtesy of Linda Levi; Figure 2–7, Chapter 3, Figure 3–4, Chapter 5, Chapter 6, Figure 6–2, Figure 6–9, Chapter 7, and Chapter 9 photos courtesy of Susan Empson; Figure 6–1 photo courtesy of Julian Joseph.

The research and writing of this book was supported in part by grant number 0138877 from the National Science Foundation and a Faculty Research Award from The University of Texas at Austin.

For information regarding teacher professional development on the material presented in this book, please contact Linda Levi at linda.levi@teachersdg.org or 877-650-1914.

Library of Congress Cataloging-in-Publication Data

Empson, Susan B.

Extending children's mathematics : fractions and decimals / Susan B. Empson and Linda Levi.

p. cm.

Includes bibliographical references and index.

ISBN-13: 978-0-325-03053-1

ISBN-10: 0-325-03053-7

1. Fractions—Study and teaching (Elementary). 2. Decimal fractions—Study and teaching (Elementary). 3. Fractions—Study and teaching (Middle school). 4. Decimal fractions—Study and teaching (Middle school). I. Levi, Linda. II. Title.

QA117.E47 2011

372.7′2—dc22

2011001314

Editor: Victoria Merecki
Production editor: Sonja S. Chapman
Typesetter: Publishers' Design and Production Services, Inc.
Cover and interior designs: Bernadette Skok
Author photo of Linda Levi courtesy of Katrin Talbot
Author photo of Susan Empson courtesy of Robert Donald
Cover photo courtesy of Susan Empson
Manufacturing: Steve Bernier

Printed in the United States of America on acid-free paper

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For the second year in a row, Ms. Perez is teaching fractions based upon the framework for problem types and solution strategies provided in this book. She started this unit on fractions by having her students explore Equal Sharing and Multiple Groups problems. On this day, she teaches a lesson in which she plans to highlight and allow students to reflect as a class upon the Relational Thinking in a couple of students' strategies. She provides some background and then explains her thinking as she was planning and teaching this lesson.

Ms. Perez's Fifth-Grade Lesson

My fifth-grade class has been studying fractions for about six lessons. Although the students had a unit on fractions last year, in many ways this was the first time they have really been pushed to think about the deep ideas of fractions. At the beginning of the unit, a lot of my students didn't think about fractions as numbers and most of them didn't connect fractions with what they knew about division of whole numbers.

Getting Started

I started fraction instruction by having students solve Equal Sharing problems. Because we had already established a routine for solving and discussing problems and my students were used to tackling problems on their own, I didn't show them what to do. I just used a context they were familiar with and told them that I wanted them to solve the problem in any way that made sense to them. I try to make time each day to let students share their strategies so they can learn from each other. All of my students were able to solve these problems, although some of them struggled at first with the conventional names for the resulting share. After working with Equal Sharing problems, all of my students were able to correctly name fractional amounts for mixed numbers and for fractions less than one, and they understood that fractions were numbers.

I then moved on to Multiple Groups problems. I started with multiplication problems with a unit-fraction amount in each group. For example, we solved the problem:

> I AM MAKING SUB SANDWICHES for some friends. There will be 13 of us eating sub sandwiches. I want to serve each person ¼ of a sub sandwich. How many sub sandwiches do I need to make all together?

Many of my students used Relational Thinking strategies to solve this problem. For example, they would say "4 one-fourths are a whole, so 8 one-fourths would be 2 sandwiches, and 12 one-fourths would be 3 sandwiches, and then you only need another one-fourth of a sub. You would need $3\frac{1}{4}$ sandwiches to feed your friends." After working with several Multiple Groups problems—both Multiplication and Measurement Division—where students were combining unit fractions, we moved on to this multiplication problem with groups of nonunit fractions:

> MR. DAVIS IS PLANNING an art project for his class. Each student will need $\frac{3}{4}$ of a package of clay to do this project. If Mr. Davis has 12 students in his class, how many packages of clay would he need?

My first goal in giving this problem was for each and every student to solve the problem using a strategy that made sense to him or her. My second goal was to find some students who used Relational Thinking strategies and have them share those strategies with the class. I knew a lot of students would either draw out every $\frac{3}{4}$ package of clay or repeatedly add three-fourths. Although these are valid strategies for solving this problem, I was hoping to find some students who used Relational Thinking. For example, maybe they would see that 2 groups of $\frac{3}{4}$ was $1\frac{1}{2}$ and then combine groups of $1\frac{1}{2}$ rather than groups of $\frac{3}{4}$. I was also hoping that someone might even recognize that 4 groups of $\frac{3}{4}$ was 3 and then be able to quickly combine groups of 3 while still keeping track that 3 packages of clay would represent what 4 students would get. When the students who used Relational Thinking shared their strategies with the class, I planned to introduce equations to help other students understand the Relational Thinking.

I started the lesson by reading the problem to the class and giving each student a sheet of paper with the problem written on it. I walked around the room as the students were solving the problem and mostly just watched what they were doing. If students were frustrated or stuck, I asked if they needed help. Sometimes they said no and I just moved on. If they said yes, I asked them to tell me the story in their own words. If they couldn't retell the story, we worked on understanding what was happening in the problem. I have some struggling readers and English language learners who often need some support to understand what is happening in the problem. I started by checking in with these students to see if they understood the story. I asked them to show me how much 1 person would get. Some students drew ¾ of a package of clay and others just wrote ¾. I then said, "Can you show me what would 2 people get?" I didn't need any other probes today to help students get started. For my students who really had to work to show me what 2 people would get, I asked them to solve the problem for 6 students rather than 12. For students who solved the problem quickly, I asked them to figure out how much clay would be needed for 21 students rather than 12.

Checking in on Students' Strategies

Once everyone was started working, I began to look for evidence of Relational Thinking. I noticed that Sonya and Lamar used Relational Thinking.

Figure 5–1 shows Sonya's paper. When I asked Sonya to explain to me what she had done, she said, "¾ plus ¾ is what 2 kids would get. I knew that was 1½. It is really easy to add 1½ and 1½. That is how much 4 kids would get and it is 3 packages. (She pointed to the numbers in the second row as she said this.) Then another 1½ packages would be 2 more kids, so 6 kids, and that is 4½ packages. I just kept adding 1½ packages, which is always for 2 more kids, and kept figuring out how much clay that would be. I stopped when I had 12 kids." Her way of writing her strategy was confusing, but her strategy made sense and used Relational Thinking.

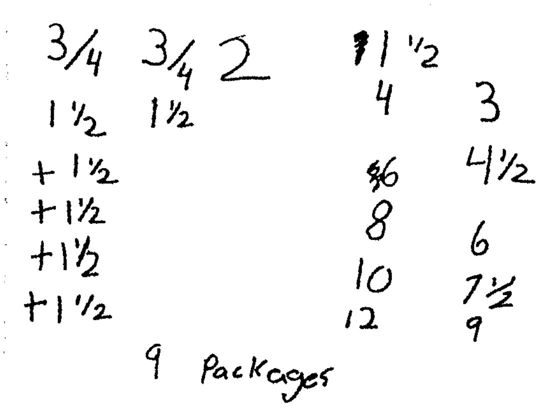

Figure 5–1. *Sonya's strategy for 12 groups of ¾*

Lamar's paper is shown in Figure 5–2. When I asked him to explain his strategy, he said, "¾ plus ¾ is 1½, so 1½ packages is 2 people, double that, 3 packages is 4 people, double that, 6 packages is 8 people, but if I double again, I get too many people. I need clay for 4 more people, so 3 more packages. 6 plus 3

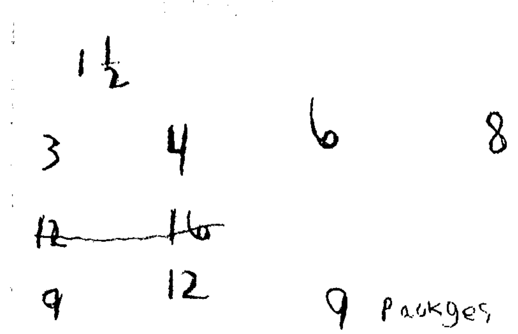

Figure 5–2. *Lamar's strategy for 12 groups of ¾*

is 9, so 9 packages." Lamar used a more efficient Relational Thinking strategy than Sonya. Again, his way of writing his strategy was pretty confusing, but his strategy made sense.

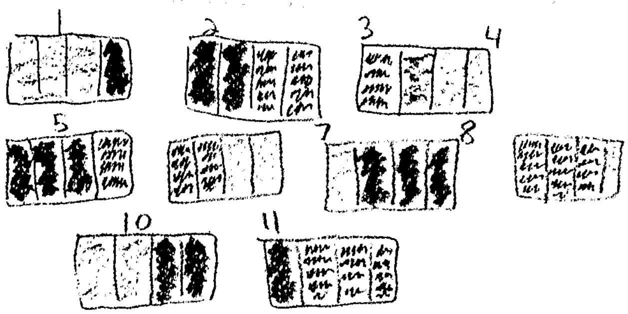

Figure 5–3. *Alex's strategy for 12 groups of ¾*

Sharing Strategies: Helping Students Make Connections

After the students had been working on solving this problem for about twenty minutes, I got their attention and told them that we were going to listen to how each other solved the problem. Alex solved the problem by drawing out groups of ¾ and I had him share first (Figure 5–3 shows what Alex drew on the board). I wanted to start with a basic strategy like his because all students would be able to understand it, and I wanted a strategy pictured so that we could refer back to it when the Relational Thinking strategies were shared.

Alex said, "I wasn't sure how many packages we would need so I just started drawing packages and cutting them into fourths. I got kinda tired of drawing packages. I went back and colored in what one kid would get—see here is what one kid would get—then here is what another kid would get, then here is what another kid would get. And then I thought, I need to remember how many kids are getting clay. That is when I put the numbers on the top. When I got to 12, I knew I could stop. All I had to do was count the packages. I got 9."

At this point, I was thinking about the students like Lucas who had struggled with the problem and I wanted to make sure they understood Alex's solution. Lucas had drawn out separate groups of ¾ and gotten an answer of 36/4. I had Lucas go up to Alex's drawing and show me the clay that Alex gave the first person, and then the clay Alex gave the second person. Since Alex used color, Lucas was easily able to do this. I asked Lucas, "When Alex gave clay to 2 people, how many packages did he use?" At first Lucas answered six-fourths. Through conversation and input from other students, Lucas was able to see that this could also be one and two-fourths or $1\frac{1}{2}$ packages.

After discussing Alex's strategy, we moved on to Sonya. I wanted Sonya to share her strategy before Lamar because I thought it would be easier to understand than Lamar's. Since the way that Sonya wrote her strategy was pretty confusing, my plan was to introduce equations to help others understand her strategy. These equations would also help us focus on the relationships that Sonya used.

Ms. Perez: Sonya is going to share next. Sonya, I'm not going to have you show us what you wrote on your paper. I want you to tell us how you solved the problem. I am going to be the recorder, and I am going to write what you did using equations. Everyone is going to have to listen and watch really carefully because I want to make sure that what I write matches how Sonya solved the problem. OK, Sonya, what did you do first?
Sonya: First I knew that $\frac{3}{4}$ and $\frac{3}{4}$ is $1\frac{1}{2}$. That is how much 2 kids would get.
Ms. Perez: She said $\frac{3}{4}$ *and* $\frac{3}{4}$. Does anyone have an idea how we could write that in an equation? Peter?
Peter: She added them, $\frac{3}{4}$ plus $\frac{3}{4}$ is $1\frac{1}{2}$.
Ms. Perez: OK, does this equation [writing on the board] show what Sonya did?

$$\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$$

Students: Yes.
Sonya: But that was 2 kids, you need to write *2* or you might forget.
Ms. Perez: Good point, should we write *2 kids* next to this?

$$\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2} \qquad \text{2 kids}$$

Sonya: Yes, you can't forget that that is how much for 2 kids.
Ms. Perez: OK, so we have 2 kids would use $1\frac{1}{2}$ packages of clay. Claire, can you go up to Alex's picture and show me where he shows that 2 kids would get $1\frac{1}{2}$ packages of clay? [Claire goes up the board and uses her finger to circle the first package and a half of Alex's picture.] Claire, what number does Alex have up there by that part of his strategy?

Claire: *2*, for 2 *kids*.

Ms. Perez: OK, Sonya, what did you do next?

Sonya: $1\frac{1}{2}$ and $1\frac{1}{2}$ is really easy for me to add. It is 3. So I got 3 packages, but I had to remember that 3 packages is for 4 kids.

Ms. Perez: OK, let's see if this equation represents what you just told me:

$$1\frac{1}{2} + 1\frac{1}{2} = 3$$

Sonya: But you need the *4*, you can't forget the 4 kids.

Ms. Perez: OK, let me write that.

$$1\frac{1}{2} + 1\frac{1}{2} = 3 \quad \text{4 kids}$$

Ms. Perez: Is that what you have done so far?

Sonya: Yes.

Ms. Perez: Alex, did you also find that 4 kids would use 3 packages?

Alex: Yes, I did [walks to the board]. Here is where I wrote the 4 kids and look, they get 3 packages of clay [with his finger, circles the first 3 packages of clay and the numeral *4* above it].

It is really important to have kids like Alex make connections between the more basic strategy and the Relational Thinking strategy. I need to ask questions to students to help them think about the strategies that other students are sharing. Even though my questions help students listen and pay attention, the more important purpose behind them is to help students draw connections between the strategies. This helps students to see the relationships and maybe even to use the relationships in their next strategy.

Next I wanted to focus on how Sonya moved from 2 kids using $1\frac{1}{2}$ packages of clay right to 4 kids using 3 packages of clay without needing to figure out how much clay 3 kids would need. Many students didn't understand what she did. I asked questions of students such as Terry and Peter, because I thought they were close to understanding, and with a little additional support, could understand the strategy.

Ms. Perez: Sonya went right from 2 kids to 4 kids. She never figured out how much clay for 3 kids. With Alex's strategy, I saw how much 3 kids would get but I don't see it here with Sonya's strategy. Terry, can you tell me what you think she did?

Terry: I am confused. No one gets $1\frac{1}{2}$ packages, they each get $\frac{3}{4}$.

Sonya: No, but 2 kids get $1\frac{1}{2}$ packages so 4 kids get 3 packages. It is easier to add $1\frac{1}{2}$ than to add $\frac{3}{4}$.

Terry: I still don't get it. I got 4 kids get 3 packages too, but I added $\frac{3}{4}$ 4 times.

Tyrone: I think I get it. It's like you double both: if 2 kids get $1\frac{1}{2}$ packages, the 4 kids get double what 2 kids get, they get 3 packages.

Terry: [long pause] Oh, you double both, kids and packages. Oh, that is a good idea!

Ms. Perez: Sonya, what did you do next?

Sonya: I didn't add another $\frac{3}{4}$, I added another $1\frac{1}{2}$, 4 plus $1\frac{1}{2}$ is $5\frac{1}{2}$, that is 6 kids.

Ms. Perez: Terry, what do you think about this? She didn't add another $\frac{3}{4}$.

Terry: No, she adds what 2 more kids would get, that is a good idea!

Ms. Perez: Peter, what do you think is the difference between what Terry did and what Sonya did?

Peter: I did it like Terry, but I think I get what Sonya did. It is easier to add one and one-halves than three-quarters, so she added what 2 kids would get each time.

I was pretty sure that Alex, Terry, and Peter now had an intuitive grasp of Sonya's strategy. The conversation probably helped other students to gain some understanding too. Now I focused on the equations that could represent Sonya's strategy.

Ms. Perez: Sonya, do you think you could write the next step with an equation? [Sonya writes at board:]

$4 + 1\frac{1}{2} = 5\frac{1}{2}$ 6 kids

Terry: She did it again, she added what 2 kids would get.

Lamar: I thought she would double the whole thing. That's what I did.

Ms. Perez: Lamar, we will look at yours soon, but let's finish Sonya's here.

We continued with Sonya's strategy until we had the following written on the board:

$\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$	2 kids
$1\frac{1}{2} + 1\frac{1}{2} = 3$	4 kids
$3 + 1\frac{1}{2} = 4\frac{1}{2}$	6 kids
$4\frac{1}{2} + 1\frac{1}{2} = 6$	8 kids
$6 + 1\frac{1}{2} = 7\frac{1}{2}$	10 kids
$7\frac{1}{2} + 1\frac{1}{2} = 9$	12 kids

If Sonya had shared without my writing anything down, only a few students would have been able to understand her strategy. Not only did these equations help other students understand her strategy, they also helped us focus on the mathematical relationships she used. Students need to learn how to write equations to show their ideas. My goal is for students to connect equations to strategies and problem situations and to eventually be able to use equations to communicate mathematical ideas.

My work introducing equations to represent solution strategies reminds me of my literacy instruction when I taught first grade. My first graders had all these ideas and my role was to introduce conventions that would help them communicate their ideas. For example, I taught them about spacing between words, capital letters, periods to mark off sentences, some standard spellings, and so on. It is much the same when I introduce equations to these older students. They have these great ways of solving problems, and my job is to introduce conventions so that they can represent and communicate these ideas.

Next I asked Lamar to share his strategy. I chose not to have him share what he wrote because I wanted to continue to introduce equations to represent solution strategies, and I knew that equations would focus on relationships in a way that Lamar's notation did not. I started by having Lamar explain how he solved the problem. He is a student who needs to get his whole idea out there, so I didn't interrupt him with notation right away.

Ms. Perez: Lamar, could you share how you solved this problem?

Lamar: I did $\frac{3}{4}$ plus $\frac{3}{4}$ is $1\frac{1}{2}$. $1\frac{1}{2}$ packages would be what 2 people would get. If I double it, I have 3 packages and 4 people. If I double it again, I have 6 packages is 8 people. I tried to double the whole thing and I got 16 people and 12 packages, but that is too many people. With 6 people and 8, I need clay for 4 more people, so 3 more packages. 6 plus 3 is 9, so 9 packages for 12 people.

Ms. Perez: OK, can someone tell me what Lamar did?

Terry: It was too fast!

Ms. Perez: Yes, it was fast. Sometimes it is hard to communicate our ideas only with talking. Today, Alex used a picture to help us understand how he solved the problem, and when Sonya shared, we wrote equations to help us understand how she solved the problem. We are going to try to write equations to help us understand Lamar's strategy. Lamar, why don't you start again? This time I am going to try to write some equations to show what you did. Everyone needs to listen and watch really closely because I want to make sure that what I write matches how Lamar solved the problem. OK, Lamar, why don't you tell us the first part about what you did?

Lamar: First, I found that 2 kids would need $1\frac{1}{2}$ packages, $\frac{3}{4}$ plus $\frac{3}{4}$ is $1\frac{1}{2}$.

Ms. Perez: OK, does this equation show what you did at first?

$$\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$$

Lamar: Yes.

Ms. Perez: How about the rest of you, does this show what Lamar just said?

Sonya: What about the 2? He said there were *2* packages and you don't have that.

Ms. Perez: Good point, I think we had this same problem when we started with yours. How can we write an equation where we show that Lamar had 2 groups of $\frac{3}{4}$?

Claire: You could write *2 kids* like we did with Sonya.

Ms. Perez: Yes, I could do that. I am going to challenge you here though. Is there a way that I could write an equation with a *2* in it that shows what he did?

Since Sonya's strategy was about adding what 2 students would get each time, I didn't feel compelled to get the students to write a multiplication equation for her strategy. Lamar did a lot of doubling and the best way to show doubling is with multiplication equations. I was hoping that we would be able to write a multiplication equation, so I didn't accept the suggestion that we write *2 people* off to the side like I did with Sonya.

Peter: It isn't $\frac{3}{4}$ plus 2. That isn't right.

Lamar: No, I didn't add 2, I did 2 kids.

Ms. Perez: And how much did each kid get?

Lamar: $\frac{3}{4}$ package.

Ms. Perez: So you have 2 kids and each gets $\frac{3}{4}$. Can anyone think of an equation that would go with 2 kids, each getting $\frac{3}{4}$? [Long pause in which no one volunteers.] What if instead of $\frac{3}{4}$ of a package each kid got 5 packages, can you think of how we could use an equation to show that 2 kids each get 5 packages?

Tyrone: We talked about this before, it could be 5 plus 5, but it could also be 2 times 5—2 groups of 5.

Ms. Perez: [writes *2 × 5* on the board] Tyrone says that this shows 2 groups of 5. Now with the problem that you all just solved, did we have 5 packages for each student?

Chorus: NO!

Claire: We had $\frac{3}{4}$ for each kid.

Ms. Perez: How could we write 2 groups of $\frac{3}{4}$? Remember, this one shows 2 groups of 5.

Tyrone: How about 2 times $\frac{3}{4}$?

Ms. Perez: [writes $2 \times \frac{3}{4}$] How does that look for a start? What does this mean? Lamar, what does this mean?

Lamar: 2 groups of $\frac{3}{4}$. That is what I did, 2 groups of $\frac{3}{4}$ is $1\frac{1}{2}$.

Ms. Perez: So is this a way of writing Lamar's first step?

$$2 \times \frac{3}{4} = 1\frac{1}{2}$$

We discussed for quite some time why this equation fit with what Lamar did. I knew this equation would challenge a lot of my students since many of them entered fifth grade writing addition equations for simple multiplication story problems. For example, if the problem was something like, "I have 5 buckets with 43 marbles in each bucket, how many marbles do I have?" they would write:

$$43 + 43 + 43 + 43 + 43 = n$$

as the equation that goes with this problem. Of course this equation isn't wrong, but I wanted them to know that

$$5 \times 43 = n$$

also represents with this problem. If they have a problem like 87 buckets with 43 marbles in each bucket, the only practical equation they can write to go with this problem is a multiplication equation. Almost all of my students can now write multiplication and division equations to represent story problems with whole numbers, but writing multiplication and division equations to represent situations that involve fractions is a new challenge for them.

The Discussion Continues

Ms. Perez: OK, Lamar, let's get back to your strategy. You started with 2 groups of $\frac{3}{4}$ is $1\frac{1}{2}$. Then what did you do?

Lamar: Then I just doubled it. If 2 kids is $1\frac{1}{2}$ packages, then 4 kids would be 3 packages. I wrote *3* and *4* on my paper.

Ms. Perez: So, I am wondering if there is an equation we could write that would show this idea, that 4 kids would get 3 packages of clay.

Lamar: It is just like the one you wrote, but this time you have 4 groups of $\frac{3}{4}$ is the same as 3.

Ms. Perez: Do you want to come up and write it?

Lamar: Sure.

Ms. Perez: Write it here, right under where we wrote the other equation.

Lamar: OK.

$$2 \times \frac{3}{4} = 1\frac{1}{2}$$
$$4 \times \frac{3}{4} = 3$$

Ms. Perez: OK, can someone read what Lamar just wrote and tell me what it means?

Terry: 4 groups of $\frac{3}{4}$ is 3. He said he doubled both, the kids and the clay.

Ms. Perez: Does that work?

Terry: Yeah, if 2 kids have $1\frac{1}{2}$, 4 kids will have 3. You double both.

Ms. Perez: Tyrone, what do you think?

Tyrone: I think it's right—4 groups of $\frac{3}{4}$ is 3. Double the kids and double the packages.

Ms. Perez: OK, then what did you do next?

Lamar: Hmm. I don't remember what I already told you and what I didn't tell you yet.

Ms. Perez: Sometimes it is hard to remember just one part of how you solved the problem. See if you can use the equations that we are writing to help you remember where you are.

Lamar: Oh, yeah, I doubled it one time, then I doubled it again, if 4 kids would need 3 packages, then 8 kids would need 6 packages.

I was happy that Lamar could look at the equations we had written and figure out what he had already explained to us. I hope eventually students will see that equations not only help us communicate with others but also can help us solve problems. Many students already use equations to solve problems with whole numbers. My goal is for them to be able to transfer this to their work with fractions.

Ms. Perez: OK, how about someone else, can someone else tell me what the equation for this part would be? Alex, do you have an idea?

Alex: It has to have an 8 and a 6.

Ms. Perez: Yes, it does. Do you remember what Lamar said about the 8 and the 6?

Alex: 8 kids would need 6 packages?

Ms. Perez: Yes, and how much does each kid get?

Alex: $\frac{3}{4}$ of a package.

Ms. Perez: OK, so 8 kids each get $\frac{3}{4}$ of a package and that is 6 packages. Is this right?

Alex: Yes.

Ms. Perez: And how could we write that in an equation?

Alex: I am not sure.

Jessie: I have an idea. [Comes to the board and writes.]

$8 \times 3/4 = 6$

Ms. Perez: Why do you think that would work, Jessie?

Jessie: Because 8 groups of 3/4 is 6. That is what Lamar said.

Ms. Perez: OK, what do you think, Alex?

Alex: That is probably right.

Ms. Perez: OK, Lamar, what did you do next?

Lamar: [long pause, looking at the equations on the board] OK, I see where I am. I doubled it again. If 8 kids get 6 packages, then 16 kids get 12 packages, but that is too many kids so I crossed it off.

Ms. Perez: OK, Does this show what you did?

$16 \times 3/4 = 12$

Lamar: Yes, but that was too many kids, so I went back to 8 kids and 6 packages. I only need 4 more kids to get 12 kids and I already know that 4 kids need 3 packages, so I just kind of added them together and got 12 kids would need 9 packages.

Ms. Perez: You just said a lot. Let's break that down a little. Look at this equation [pointing], $16 \times 3/4 = 12$. Why did Lamar say that he couldn't use this?

Peter: That is for 16 kids. It's too many kids.

Ms. Perez: OK, so what did he do?

Peter: He said he went back to 8 kids need 6 packages.

Ms. Perez: [points to $8 \times 3/4 = 6$] OK, so he went back to there.

Peter: Yeah and he added 4 more kids. Oh, he has that up there! 4 times 3/4 is 3.

Lamar: It is kinda like I added two equations together.

Ms. Perez: Let's look at that:

$4 \times 3/4 = 3$
$8 \times 3/4 = 6$

You added those together?

Lamar: Yeah, kinda, the way I think about it is, I had 4 groups of 3/4 and then 8 groups of 3/4, and that is like 12 groups of 3/4 altogether.

Ms. Perez: Now that is interesting. What do the rest of you think? Would 4 groups of ¾ plus 8 groups of ¾ be the same as 12 groups of ¾?

Peter: It would have to be!

Terry: I need to hear it again.

Ms. Perez: Would 4 groups of ¾ plus 8 groups of ¾ be the same as 12 groups of ¾?

Terry: Yes, it would, it is like what 4 kids get plus what 8 kids get would be the same as what 12 kids get!

Ms. Perez: I am going to write this idea in an equation to see if that helps more people think about it.

$$4 \times 3/4 + 8 \times 3/4 = 12 \times 3/4$$

Ms. Perez: I know this is a long equation here but let's look at it together. 4 groups of ¾ plus 8 groups of ¾ is the same as 12 groups of ¾. Is that true?

Lamar: Yes, and that is what I did.

Sonya: Oh, I get it now! He did 4 people's clay and 8 people's clay is 12 people's clay. That's right.

Ms. Perez: So, Lamar, does this show what you did?

Lamar: No, I need to show the answer, I need to show the 9 packages.

Ms. Perez: Oh, of course you do. I got so excited about this equation that I forgot about that! How could you show it?

Lamar: Could I just write = 9 at the end? Can you have two equal signs?

Ms. Perez: Actually you can, why don't you do that?

$$4 \times 3/4 + 8 \times 3/4 = 12 \times 3/4 = 9$$

Figure 5–4 shows what our board looked like.

I was really excited that Lamar said something about adding equations together. Although that isn't one of our standards at fifth grade, I know that in high school algebra, students will eventually need to understand how they can add equations together. I was also excited that the equation we wrote showed the distributive property with a fraction amount in each group.

That was a really complex equation and several students were able to talk about it. It was my sense at this point that about a third of the class understood this equation. When we work with complex ideas like this, I don't expect everyone to understand them the first time. And you never really know what small seeds are planted for some students who appear not to understand at all. We will return to this idea often, and I expect everyone to grow in their understanding of these equations.

$$2 \times \frac{3}{4} = 1\frac{1}{2}$$
$$4 \times \frac{3}{4} = 3$$
$$8 \times \frac{3}{4} = 6$$
$$\require{cancel}\cancel{16 \times \frac{3}{4} = 12}$$
$$4 \times \frac{3}{4} + 8 \times \frac{3}{4} = 12 \times \frac{3}{4} = 9$$

Figure 5–4. *The board after Ms. Perez and Lamar finished writing equations to represent his strategy*

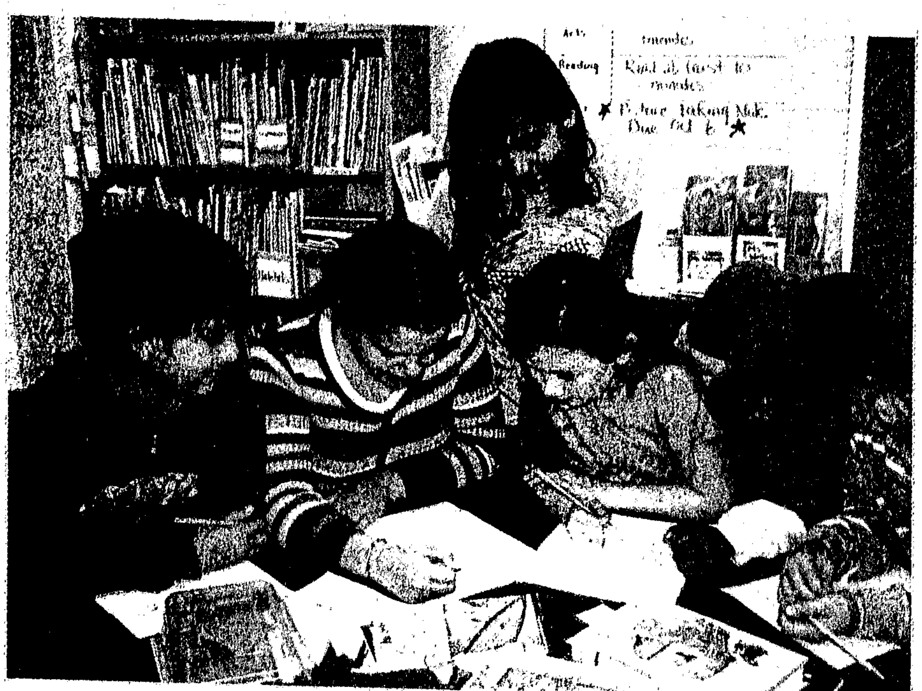

Figure 5–5. *Students discuss their strategies*