

CHAPTER TWELVE

HOW MUCH IS BLUE?

A PATTERN BLOCK ACTIVITY

Overview

In this activity, students are shown a design made with thirteen pattern block pieces—three green triangles, six blue parallelograms, three red trapezoids, and one yellow hexagon. The problem they solve is: “What fractional part of this design is blue?” The challenge of this problem for students is that they must think about both the fractional parts of the total design and the areas of the individual pattern block pieces. Along with finding the answer, students have to explain their reasoning.

Materials

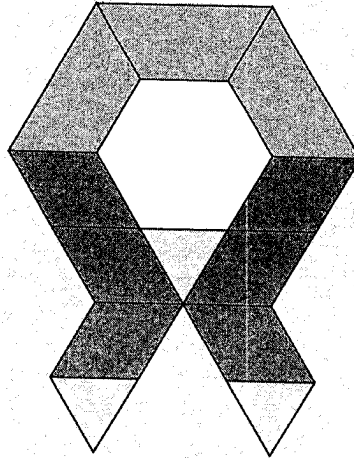
- ▲ pattern blocks, enough so that each child can build the design and have extra blocks as well, about 1 bucket per 6 students
- ▲ *Pattern Block Design* worksheet, 1 per student (see Blackline Masters)
- ▲ optional for extension: *Pattern Block Triangle Paper* worksheet, 1 per student (see Blackline Masters)

Time

- ▲ two class periods, plus extra time for students to solve one another's puzzles

Teaching Directions

1. On chart paper or on an overhead transparency, draw the design as shown (see next page) so all students can see it.
2. Discuss with the students what they know about pattern blocks. Then write on the board the problem the students are to solve: *What fractional part of this design is blue?* Ask the children to think about the problem quietly by themselves before you begin a class discussion.



3. After a few moments, begin a class discussion by asking: "Who has an estimate of what the answer might be?" Discuss their ideas.
4. Then give each student a copy of the design drawn to the actual size of pattern blocks. (See Blackline Masters.) Also, distribute pattern blocks. Give the class directions: "Prepare your papers by writing the question at the top. Figure out the answer by using the blocks, drawing pictures, or doing whatever will help you. Then write about how you reasoned." If you have students work in pairs, it's still a good idea for each to write an individual paper.
5. Circulate as the students work, giving help as needed.
6. The next day, have students who are willing report how they solved the problem. To extend the experience, give the following directions: "Each of you should make your own design using an assortment of green, blue, red, and yellow pattern blocks. Draw your design on pattern block paper and then pose a question: What fractional part of the design is _____? (Choose one of the colors.) On a separate sheet, record the answer and explain your reasoning."
7. Check the students' papers for accuracy. Then either devote class time for all students to figure out answers to one another's designs or organize the designs and answers into two folders and use them for a choice activity.

Teaching Notes

In the November 1995 issue of the NCTM journal *Teaching Children Mathematics* (Volume 2, Number 3), Janet H. Caldwell, professor of mathematics at Rowan College in New Jersey, wrote an article titled "Communicating About Fractions with Pattern Blocks." After reading the article, I tried the activity and found it effective for engaging students and assessing their understanding.

An interesting aspect of the problem is that while six of the thirteen blocks are blue, the blocks of different colors are each different sizes. Therefore, it's not true that six-thirteenths of the design is blue. In order to solve this problem correctly, students have to consider the area of each of the pieces.

For this activity, the students should have access to pattern blocks. As with all manipulative materials, it's important that students are comfortable and familiar with the blocks before being asked to focus on this problem. They should have had prior experience exploring pattern blocks and the relationships among them. If this is not the case, then I recommend first trying the introductory activities presented in Chapter 4, "Exploring Fractions with Pattern Blocks."

The Lesson



DAY 1

To prepare for the lesson, I drew on chart paper an enlarged version of the design made from pattern blocks and posted it for all of the students to see.

"It's a rocket!" Sam said.

"Maybe it's a kite," Michael added.

I didn't comment on the boys' remarks but addressed the class. "What do you know about pattern blocks?" I asked. The students were familiar with pattern blocks and I wanted to focus their attention on them before presenting the problem.

"Two greens make a blue," Emma said.

"And six greens make a yellow," Nick added.

"You can make a yellow in lots of different ways," Sean said.

"Not so many ways," Amy responded.

"Just six, I think. Or maybe seven." Earlier we had investigated the different combinations of red, blue, and green pattern blocks that could be used to build the yellow hexagon. Then, considering the hexagon as the whole, we assigned fractional values to each of the other pieces. (See the *Exploring Fractions with Pattern Blocks* lesson, pages 39–45.)

"I know the names of the shapes," Alma said. She then correctly recited them, "Triangle, parallelogram, hexagon, and trapezoid."

"I remember that you can use a red one and three triangles to make the yellow hexagon," Ramon said.

Some hands were still raised, but I said, "You seem to know about the pattern

blocks, so rather than hearing more ideas, let me tell you about the problem I'd like you to solve."

I directed their attention to the chart paper. "The problem is about this design. I'll write it on the board." Writing the problem as well as giving it orally is helpful for children who don't always absorb what they hear. I read as I wrote:

What fractional part of this design is blue?

"First think about this quietly by yourself," I said. "Then we'll have a class discussion about your ideas."

The room got quiet. After a few moments, I asked the class, "Who has an estimate of what the answer might be?" About seven or eight students raised their hands immediately. I waited, and several more students raised their hands. I called on Sarah.

"I think it's a little less than half," Sarah said.

"Why do you think that?" I asked.

Sarah replied, "See, first I tried to count the blocks, but it was hard to do from here. So then I just looked at it and it looks like it's less than half." There were some murmurs of agreement from the class.

"Any other ideas?" I asked. I called on Paul.

"I counted, and I think it's six-thirteenths," Paul said.

"What did you count?" I asked.

"The blocks," Paul answered. "There are thirteen blocks and six of them are blue, so I would write six over thirteen."

Sophie chimed in, "I don't think that can be." She turned to me and asked, "Can I explain?" I nodded.

Sophie then turned to Paul. "You can't do that, Paul," she said. "You can't count all the blocks as the same because they're different sizes so they're not equal. The sizes have to be equal in fractions."

Paul thought for a moment and then said, "Oh yeah." He didn't seem convinced, however.

"Do you have an idea about an answer?" I asked Sophie.

"I'm not sure yet," she answered. "But I started figuring it out. I think it would be easier to figure with the blocks. Then I could cover the design in greens." Several children agreed.

"I was going to cover it in blues," Daniel added.

"That's what I was going to do," Sean said.

"Me, too," several others added.

"Could we try it with the blocks?" Amy asked.

The children were getting excited. I quieted them by asking for their attention and then explaining what they were to do. "I'll give each of you a copy of the design drawn to the actual size of pattern blocks. Also, I'll give a bag of pattern blocks to each group. While I do this, you can get your papers ready by each writing at the top of your paper your name, the date, and the question. Then you can figure out the answer by using the blocks, drawing pictures, or doing whatever will help you. Finally, you need to write about how you reasoned."

"Can we work together?" Tina asked.

"You can talk about your ideas with one another," I said. "But you each should write your own paper. Remember, it's fine to use an idea you get from someone else, as long as it makes sense to you and you can explain it with your own words." I distributed the pattern blocks as the students organized their papers.

Observing the Students

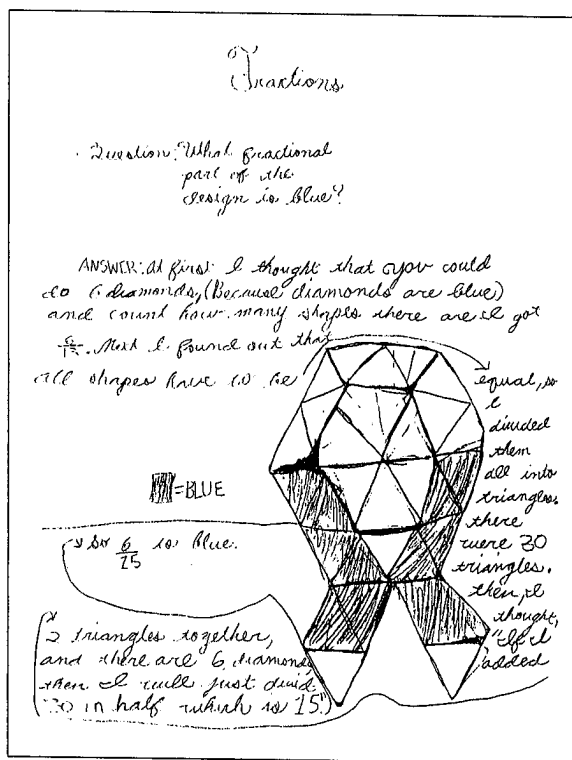
I circulated as the students worked, answering questions, prodding for more information, and refocusing students when necessary.

Paul asked me, "Should I write about what I thought first and then give another idea?"

"That would be good," I said. "Then I'll have a record of how you changed your thinking."

After first writing about the idea he had presented to the class, Paul covered the design with green triangles. He then added to his paper: *there were 30 triangles. then, I thought, "If I added 2 triangles together, and there are 6 diamonds, then I will just divid 30 in half which is 15."* So $\frac{6}{15}$ is blue. (See Figure 12-1.)

Sophie worked quickly, doing what she had said she would—covering the design



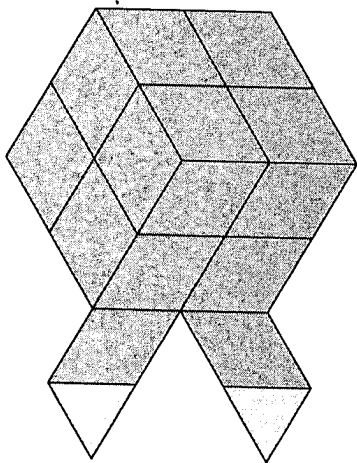
▲▲▲▲▲Figure 12-1 Paul described his original idea and then his revised idea.

with green triangles. She counted them to find that thirty triangles covered the design. Sophie wrote: *I think that it could be $\frac{12}{30}$ because if you divide all the peices into triangles there are 30, and there are twelve triangles for 6 blues.* $\frac{12}{30} = \frac{6}{15}$.

Daniel and Sean worked together to cover the design with blue parallelograms. "It doesn't work exactly," Daniel said.

"What do you mean?" I asked.

"Look," Sean explained. "You can't really cover it. You'd have to cut a blue one in half to make it really fit. But we used triangles." The boys had discovered that the shape of the design makes it impossible to cover it entirely with blue parallelograms. They had used fourteen blue parallelograms and two green triangles.

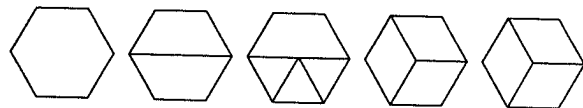


"What are you going to do now?" I asked.

"No problem," Daniel said. "It's the same as fifteen blues so the answer is six-fifteenths. We're done."

"So you're ready to write now," I said as a reminder. The boys nodded and reached for their papers.

Alma and Amy worked together and came up with an answer of two-fifths. They had rearranged the thirteen blocks in the design into five hexagons, each the same size as the yellow pattern block hexagon. The six blue blocks formed two of the hexagons.



Alma had written:

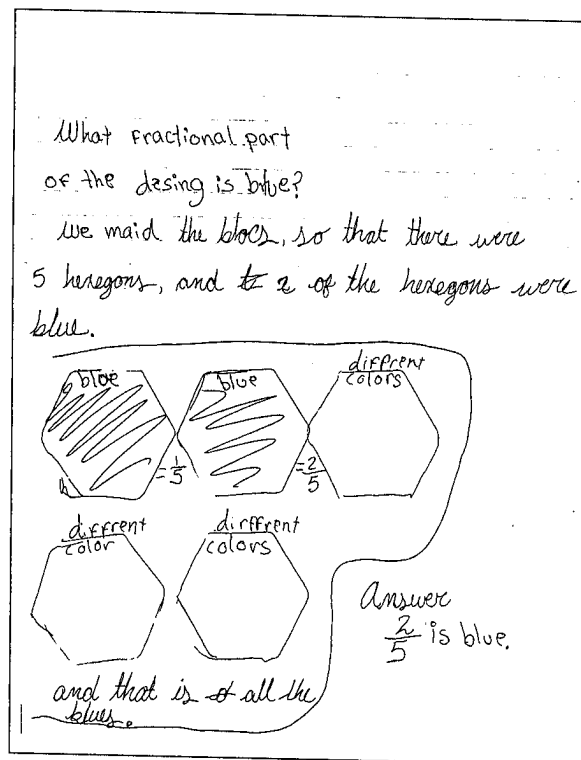
We maid the blocs so that there were 5 hexegons, and 2 of the hexegons were blue and that is all the blues. Answer $\frac{2}{5}$ is blue.

The five hexagons were on her desk. (See Figure 12-2.)

Because Sophie had finished her paper by then, I told her that Alma and Amy thought about the problem in a completely different way and figured out that the design was two-fifths blue.

"Are they right?" Sophie asked.

"Go take a look at what they did and see if you agree," I answered. Emma went with Sophie, and in a moment I heard their comments of admiration. The girls returned to



▲▲▲▲▲Figure 12-2 Alma and Amy solved the problem by rearranging the 13 blocks into 5 hexagons, using 6 blue blocks to form 2 of the hexagons.

their desks to rearrange their blocks into hexagons.

Katy brought her paper to me with a sly smile. I read what she wrote:

I say a little bit more than a third of the drawing is blue. I know this because I divided the whole drawing into diamonds. After I did that I counted the diamonds and got 15 and know that $\frac{1}{3}$ of 15 is 5 and 6 is only 1 number greater than 5.

I laughed. I enjoy when children think about something in a way I never had imagined. "Your reasoning is absolutely correct," I told Katy. "Could you also figure out what is the exact fraction that represents the blue part of the design?"

"Isn't my answer good enough?" Katy challenged me.

I answered, "It could be, depending on what the need was for figuring this out, but I'm also interested in learning as much as I can about what you know about fractions. I'm curious if you can figure out a precise answer."

Katy wasn't happy with my request, but she returned to her desk and added: *However, the exact fraction of blue diamonds is $\frac{6}{15}$.* (See Figure 12-3.)

I checked on Joey and Sarah. Together they had covered the duplicated design with

green triangles as Sophie had. Joey watched as Sarah counted the blocks.

"There are thirty," she said to Joey. "And that would be fifteen with the blue diamonds." Sarah started covering the green triangles with blue blocks, and Joey joined in when he realized what she was doing.

"So six out of the fifteen are blue," Sarah said. "That's it." She turned to write. At the top of her paper, she wrote $\frac{6}{15}$ ths. Then she explained: *We took the picture and divided all the shapes into triangles and there were 30 and as you now 2 triangles is the same as a diamond.* She drew thirty triangles and drew circles to group them into twos to show there would be fifteen diamonds.

Joey watched for a moment and then started to write: *We took the picture and we cut them all into triangles. We got 30 triangles and drew them and circled two of them and counted the circles [circles] and got 15.* He turned to Sarah for help about what to do next.

Just then, the resource teacher came for Joey. (Joey went to the resource room for extra help twice a week.) She stopped and looked at what Joey had written. "Can you take that with you, Joey?" she asked. "I'd like you to tell me about the problem you're working on. It looks interesting."

"Can I?" Joey asked me.

"Sure," I said. "And take a bag of pattern blocks, too."

Eli is a dreamy boy who enjoys drawing and writing poems and stories. He knows the words to many of the rock songs played on the radio and likes to write down the lyrics and discuss what they mean. On this day, Eli had trouble settling into the problem, and I stopped by his desk several times to help him refocus. Once Eli focused, however, he wouldn't leave for recess because he hadn't finished writing about his idea.

He wrote:

I believe the fraction is $\frac{1}{3}$ and a $\frac{1}{15}$. The reason I think this is because I did the work on paper. First I counted all the blues, I counted 6. Then I used only blues to make the whole

Pattern Blocks
I say a little bit more than a third of the drawing is blue. I know this because I divided the whole drawing into diamonds. After I did that I counted the diamonds and got 15 and know that $\frac{1}{3}$ of 15 is 5 and 6 is only 1 number greater than 5. However, the exact fraction of blue diamonds is $\frac{6}{15}$.

▲▲▲▲▲ Figure 12-3 Katy initially estimated the answer but then revised it to show the exact answer.

thing that time I counted 15 blues. I finally figured out it was $6\frac{1}{15}$ s, but that wasn't enough.

Eli then drew a circle and divided it into fifteen segments. He marked six of the segments and wrote next to the circle: $\frac{1}{3}$ and a $\frac{1}{15}$. (See Figure 12-4.)

"I'm done," he said, now eager to go outside.

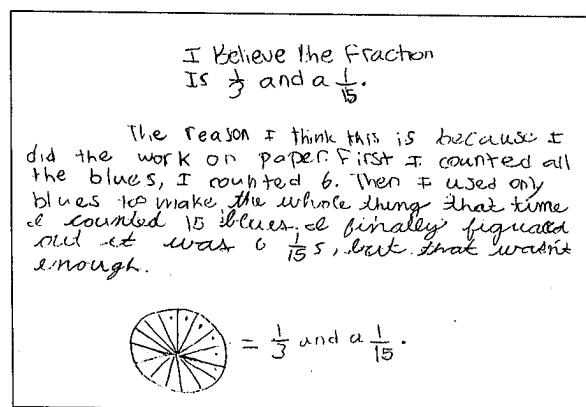
"Wait just one minute," I said. "Help me understand your idea."

Impatiently, but with confidence, Eli said, "Well, five-fifteenths makes a third, okay? But there's one piece left. And that's another fifteenth. So it's one-third and one-fifteenth. Okay?" I nodded and Eli raced to the door.

DAY 2

I began class by having students volunteer to make presentations about their ideas. It was valuable, I thought, for students to hear other ways of thinking about the problem. After the presentations, I gave directions to the class.

I said, "Today you'll each make a pattern block design of your own. I have two guidelines. One is that you must use only green, blue, red, and yellow blocks, as were used in the design I showed you yesterday. The other guideline is that your design must fit



▲▲▲▲▲Figure 12-4 Eli explains in his paper that $\frac{6}{15}$ is the same as $\frac{1}{3}$ and $\frac{1}{15}$.

on pattern block paper so you can reproduce it. It can't be larger than that." I held up a piece of the pattern block paper as a reminder; students were familiar with it from prior activities.

I continued with directions, "When you have recorded your design, write a problem at the top of the paper." I wrote on the board:

*What fractional part of the design is _____?
(Choose one of the colors.)*

"Do we have to use all four colors?" Daniel asked.

I answered, "Not necessarily."

"Can we use the orange squares and tan diamonds?" Sam wanted to know.

"No, you can't," Sophie piped up. "They don't work on the pattern block paper."

"Oh yeah," Sam remembered. Not only do the blocks not fit on the pattern block paper, but figuring their areas would be a challenge beyond the students' ability.

I gave the final instructions. "On a separate sheet, write the problem you posed, then record the answer and explain your reasoning. Before you turn in your papers, have someone try your problem to check that your answer is correct. When you check someone's paper, write your name at the bottom so that I know that the paper has been checked. Also, be sure to put your name on both your design and your answer paper." The directions seemed clear to the students and they got to work.

That night I checked their papers for accuracy. All but three were correct and I made time to talk with those students over the next two days, both the students who had created the designs and the students who had signed the papers. After all of the errors had been resolved, I put their designs into one folder and their answers in another. I showed the folders to the class.

"The problems are available as another choice activity," I explained. "You can pick any

design, solve the problem, and then find the answer in the other folder. If you can't figure out the answer, or you get an answer that's different, then talk with either the person who

created the design or the person who signed the answer sheet." I showed the class where I was putting the folders and added "Pattern Block Problems" to the choice list.

Questions and Discussion



▲ ***Why did you draw the design on a chart to begin the lesson instead of giving the students their own copies?***

I know posting the design made it difficult for the students to examine it and count the individual blocks, but I wanted to avoid having students begin work individually on the problem before we had had a class discussion.

▲ ***Why did you ask students to think quietly first before the discussion?***

While I generally allow and encourage talking in class, there are times when I think it's valuable for students to focus on a problem by themselves. This gives them a chance to collect their thoughts. I've learned that when I teach complex topics like fractions, students may understand what others say but have difficulty explaining their thinking on their own. They need ample opportunities to formulate their own ideas.

▲ ***Doesn't a class discussion give away the answer before students work on their own?***

If this were a testing situation rather than a learning opportunity, I wouldn't have the students discuss their thinking first. But I think that discussions enhance learning, which is my purpose in giving students this problem to solve. Also, students not only have to find an answer to the problem but also have to explain their reasoning. Talking and listening both help students clarify their thinking and prepare them to write. Even if students use someone else's ideas, they still have to express them in their own words.

▲ ***Why do you have students write their names on the papers they check?***

Having students sign is a way for me to be sure that someone did check each paper. Also, having to sign someone else's paper seems to help students take this part of the assignment seriously. In a way, signing makes them accountable. And for papers with errors, signatures tell me which students I need to talk with about the work.