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# Mathematics Hiding in the Nets for a

# CUBE

*Whether they are third graders or teacher candidates, students can learn about perimeter and area while having fun manipulating two-dimensional figures into three-dimensional objects.*

A common mathematical activity for geometry students is to create nets for a cube. A net is a two-dimensional figure that can be folded into a three-dimensional object. Nets help students make the connection between the concept of two-dimensional figures and three-dimensional objects. Moreover, nets help children develop a sense of the three-dimensional world.

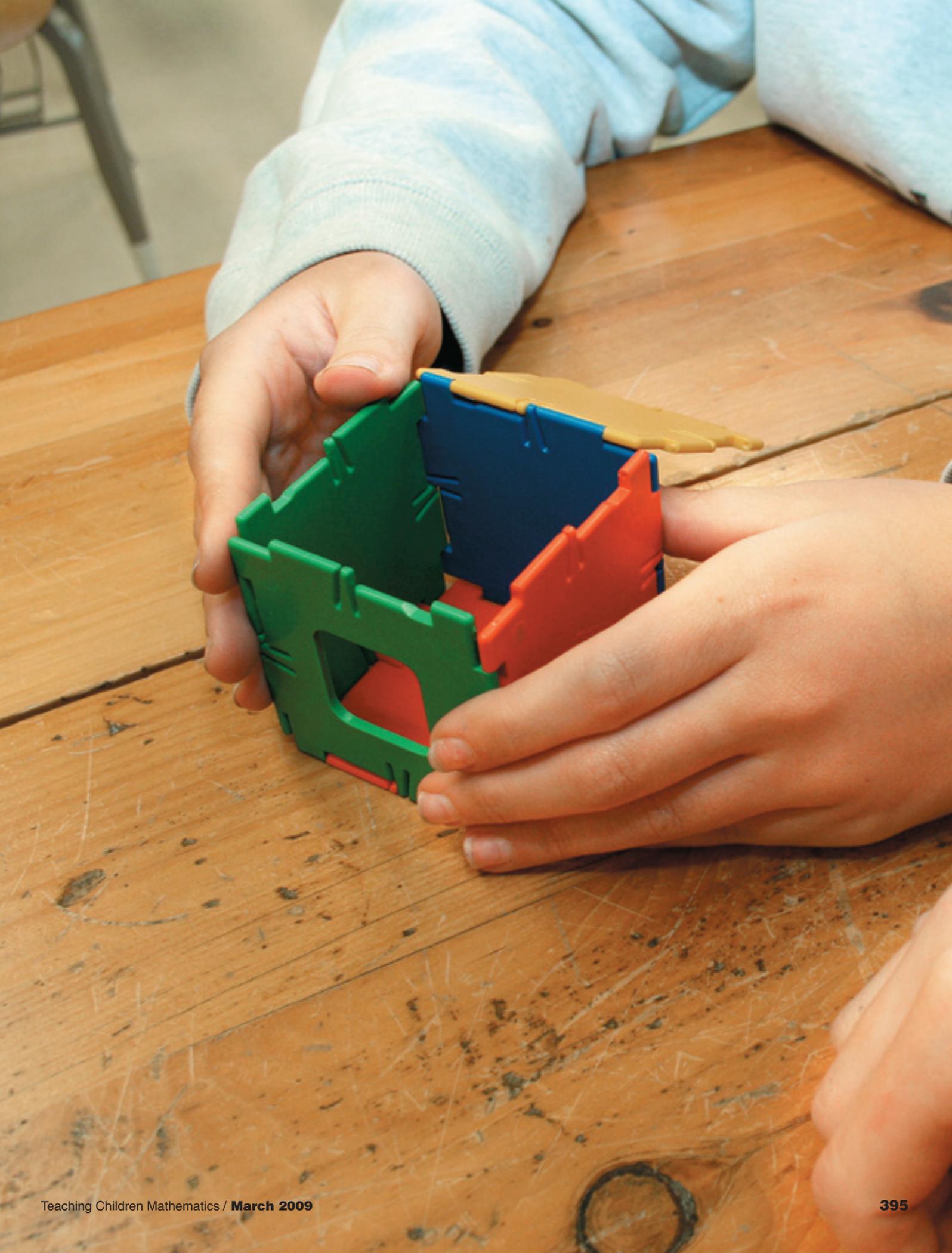
## Third Graders' Thinking

In Mann's 2004 article, "Thinking beyond the Box: Responses to the Building a Box Problem," elementary school children explored the question, "How many different nets can you draw that can fold into a cube?" The children found many different nets for a cube. They also observed common characteristics of nets for a cube. Of

By **Kyungsoon Jeon**

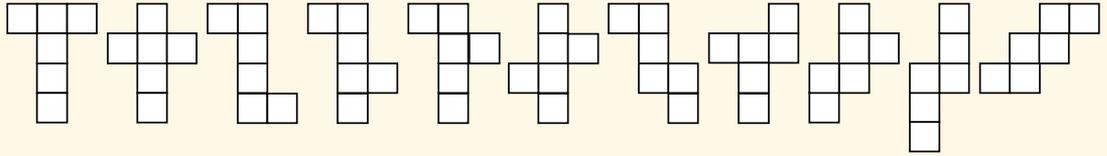


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**Figure 1**

**A cube has eleven nets.**



all their discoveries, third-grader Morgan's conclusion about cube nets is particularly intriguing:

Each net has 6 squares, and all of them have at least 4 all together and 2 on each side. If there were 3 in the middle and 3 on the side, it would not work. *All nets have a perimeter of 14 units.* They can be arranged by flips and turns. All nets start as 2-D shapes and become 3-D figures. *The area of each net is 6 square units.* The nets all have 6 square faces to make a cube [italics added]. (Mann 2004, p. 174)

The class of third graders continued investigating Morgan's claim that each net had the same area and the same perimeter. Students were amazed that although the nets were different shapes, they all had the same perimeter of fourteen units and the same area of six square units. **Figure 1** shows all possible nets for a cube, eleven noncongruent nets in total. By making connections between nets in two dimensions and cubes in three dimensions, the author of this article explains why the nets have the same area and, more interesting, the same perimeter.

## Adults' Thinking

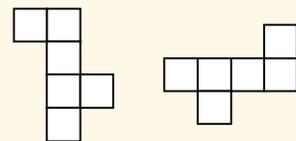
In a mathematics class for future elementary teachers, teacher candidates found the nets for a cube by using six square Polydrons™. Polydrons allow students to conjecture a net and verify if the net works by physically forming a cube from the net.

## Net exploration

During the process of conjecturing a net and forming a cube, students recorded their results on grid paper for later group discussion. Whenever a student found a new net, the professor also recorded the results on grid paper posted in the front of the room. While finding common characteristics of the nets, students engaged in a rich discussion that went beyond analyzing area and perimeter. For example, a net suggested by one student was congruent through rotation and transla-

**Figure 2**

**Are these different nets or not?**



tion to one that had already been recorded in the class data (see **fig. 2**). The class debated whether or not to consider this new net as a different net. A justification such as "rotating the first net by 90 degrees and translating will make the first one congruent to the second" helped the class redefine the project from finding *different* nets for a cube to finding *noncongruent* nets for a cube. After the discussion, students' knowledge of transformational geometry became critical for evaluating whether or not a new net would be congruent to the ones they had already found.

The Cube Nets problem provided a rich set of examples that helped students address a misconception that some of them had regarding congruency: "If two figures have the same area and the same perimeter, wouldn't they be congruent?" All the nets have the same area and the same perimeter, but they are not all congruent to one another.

## Net analysis

Why does each net have an area of six square units? An area of six square units in every net for a cube is not surprising. One student explained that a cube has six faces and that is why a net for it has an area of six square units. This reasoning is similar to that used by Morgan, the third grader: "The nets all have six square faces to make a cube."

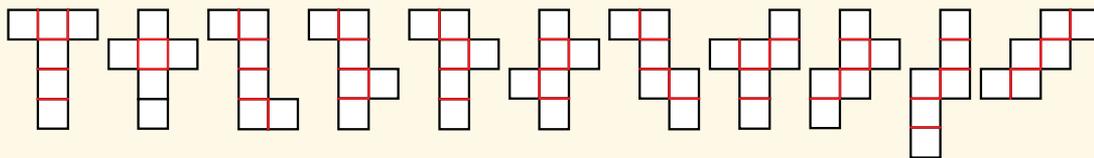
Why do all the nets for a cube have a perimeter of fourteen units? Understanding the reason is challenging to justify. For these future elementary teachers, a careful analysis of all the nets was necessary (see **fig. 3**).



During the process of finding common characteristics of the nets, student teachers engaged in a rich discussion that went beyond analyzing area and perimeter to considering congruency.

### Figure 3

The fold lines for the eleven nets for a cube are emphasized in red.

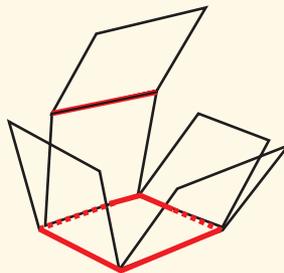


Students discovered that the fold lines are the five cube edges when a net transforms into a cube. After noticing that a cube has twelve edges in total and observing the five red line segments, a student asked where the seven remaining edges come from. This key question relates to the mathematical question, When each net transforms into a cube, what happens to each line segment that is the part of the net perimeter? Students found that each line segment would meet another line segment and form an edge of a cube (see **fig. 4**). A student explained:

One pair of the fourteen units of the perimeter joins to form an edge of the cube. Another pair of the perimeter units forms another edge, and so on. The perimeter of fourteen units, therefore, will form the seven remaining edges so that the total number of the edges of a cube is twelve (five red fold lines plus seven, half the perimeter of fourteen units).

### Figure 4

A net can be transformed into a cube.



The professor further challenged students with more questions: What happens if the perimeter is not fourteen units long? What if the perimeter is greater than or less than fourteen?

Students analyzed the two cases. If the net perimeter is greater than fourteen units, the number



A net is a two-dimensional figure that can be folded into a three-dimensional object.

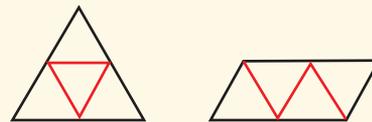
of edges formed from such a perimeter is more than seven. This means that a cube would have more than twelve edges. If the net perimeter is less than fourteen units, the number of edges formed from such a perimeter is less than seven. This means that a cube would have fewer than twelve edges. Students concluded that neither of these two cases was possible for a cube. This explains why every net for a cube has neither more than nor less than a perimeter of exactly fourteen units in addition to the five fold lines inside each net.

## From Cube Nets to Platonic Solids

Students extended the Cube Nets lesson to nets for other platonic solids. They noticed that the property of the same number of fold lines in each net and the same perimeter of every net can be extended to any

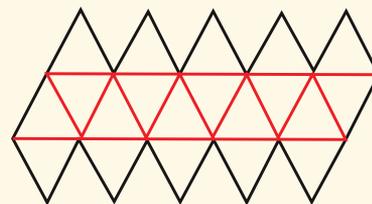
**Figure 5**

Students examine two nets for a tetrahedron and find that each net has three fold lines.



**Figure 6**

An icosahedron is a platonic solid with twenty faces.



net for platonic solids. When students examined the two nets for a tetrahedron, they found that each net has three fold lines (emphasized in red in **fig. 5**; the tetrahedron is known to have only two noncongruent nets). Each net comprises four equilateral triangles, which become faces of the tetrahedron as it folds. Both nets also have a perimeter of six units when the side length of the equilateral triangles is assumed to be the length of one unit.

When a tetrahedron net is folded, those three fold lines become three of the tetrahedron's edges. The perimeter of six units in each net forms the remaining three edges of the tetrahedron, producing a tetrahedron's six edges. On the basis of this extension, students conjectured that every net for each platonic solid has the same area in order to form the correct number of faces and also has the same perimeter in order to form the correct number of edges.

Before they folded the net and actually counted faces, edges, and vertices, the professor had students apply their conjecture to Euler's formula in order to find the number of faces, edges, and vertices for an icosahedron, a platonic solid with twenty faces (see **fig. 6**). Euler's formula is the relationship among the number of vertices ( $V$ ), edges ( $E$ ), and faces ( $F$ ) of a polyhedron ( $F + V = E + 2$ ). Together we found the following to be true:

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# More Geometry for Young Learners

1. This net is composed of twenty equilateral triangles. The number of faces of an icosahedron is, therefore, twenty ( $F = 20$ ).
2. The net has nineteen line segments within it. They will be edges of the icosahedron when folded.
3. The perimeter of the net is twenty-two units, and each pair of the twenty-two will form eleven edges when the net is folded.
4. The total number of edges of an icosahedron is thirty, since  $19 + 11 = 30$  ( $E = 30$ ).
5. Euler's formula,  $F + V = E + 2$ , then, shows that the number of vertices of an icosahedron is twelve, since  $V = E - F + 2 = 30 - 20 + 2 = 12$  ( $V = 12$ ).

This article assumes that students understand the concept of the perimeter of a two-dimensional figure, an assumption that may need to be verified by teachers of young learners. Observe carefully how students find the perimeter of a particular cube net. Once they demonstrate a strong understanding about perimeter as the distance around an object, they can more meaningfully start reasoning about why the perimeters of the nets for a cube are the same.

## Final Remarks

Classrooms where students like Morgan and her third-grade classmates actively discuss mathematical ideas and concepts are an exciting place to be. In such settings, discovery of mathematical relationships is an important goal for learning mathematics in the ways that NCTM's *Principles and Standards* (2000) advocate. Students conjecture, analyze, justify, and communicate why every net has six unit squares and why every net has a fourteen-unit perimeter. The mathematics discussed in this article helps students see why it is not just a coincidence but a significant connection that a two-dimensional net transforms into a three-dimensional cube. All cube nets must have the same area in order to form the correct number of cube faces; all cube nets must also have the same perimeter in order to form the correct number of cube edges.

## References

- Mann, Robert. "Thinking beyond the Box: Responses to the Building a Box Problem." *Teaching Children Mathematics* 11 (October 2004): 171–76.
- National Council of Teachers of Mathematics (NCTM). [illuminations.nctm.org/ActivityDetail.aspx?ID=84](http://illuminations.nctm.org/ActivityDetail.aspx?ID=84).
- . *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000. ▲

*Principles and Standards* calls for instructional programs that analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships (NCTM 2000, p. 41). The activities described in this article provide teachers with an excellent opportunity to design geometry lessons that nicely reflect the Standards. Start with an activity, for example, where your students try to find every possible net for a cube. If they are unable to find all eleven nets, have them visit NCTM's Illuminations Web site, where an applet provides twenty-four nets that may or may not form a cube. The discussion might start with the question, "How many nets for a cube are possible?"

As they use this Web site, help students justify why only eleven nets exist for a cube. A systematic analysis of the eleven nets is an authentic mathematical experience for young learners to reason and verify mathematical conjectures. Use questions to help students do an analysis:

- Which nets have four squares in the middle?
- Which ones have three squares in the middle?
- Which ones have two squares in the middle?
- And, more important, why are there only eleven possibilities?

## Challenge students

As students explore the relationship between area and perimeter in the Cube Nets problem, address another important mathematical point: Can a net with an area of six square units and a perimeter of fourteen units always be folded into a cube? This question can challenge your advanced learners even as discussions on such examples benefit the whole class in understanding the necessary but insufficient relationship: If a figure is a net for a cube, then it has an area of six square units and a perimeter of fourteen units. This is what Morgan and her classmates discussed in their search for all nets for a cube. However, the reverse is not true. The fact that a figure has an area of six square units and a perimeter of fourteen units does *not* imply that the figure will be a net for a cube. **Figure 7** shows a net with an area of six square units and a perimeter of fourteen units that will not fold into a cube.

When students work with the eleven nets for a cube, observe carefully how they find the area of a net, another fundamental concept in geometry and measurement at the elementary level. Make sure that students, especially younger ones, can communicate why counting the number of unit squares in a net gives the area. Typical questions on area depend on a formula such as width times length for a rectangle, but the irregular shapes of the nets can help students revisit the idea of area from a nontraditional perspective.

**Figure 7**

**This net with an area of six square units and a perimeter of fourteen units will *not* fold into a cube.**

