

## LESSON TRANSCRIPT

### A Demonstration Lesson:

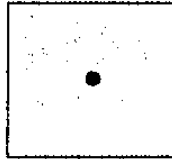
### Function Thinking in the Sixth Grade

August 3, 2000

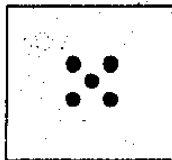
Tokyo Gakugei University, Setagaya Elementary School

Teacher: Shunji Kurosawa

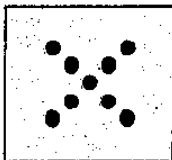
Kurosawa-sensei<sup>1</sup> began the lesson by posting a yellow sheet of paper with a single black dot on the blackboard.



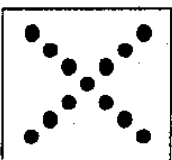
He then posted another sheet with five black dots.



This was followed by a sheet with nine black dots.



And finally, there was a sheet with thirteen black dots.



One student mentioned that it seemed to be “growing.”

Kurosawa-sensei asked students, “What is the subject of ‘growing’? *What* is growing?”

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<sup>1</sup> Sensei means teacher

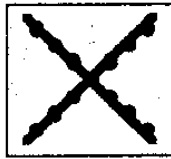
Students came up with a variety of responses. The line is getting longer.



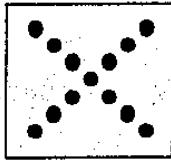
The triangular area between the lines is getting larger.



The size of the cross is getting bigger.



The number of black dots is growing.



After approximately 10 minutes of discussion, Kurosawa-sensei posed the same question again...

**Teacher:** I'll ask the question one more time.  
You have come up with some answers seeing only this one.  
(Do you see other things) by looking at this one?  
Is there something else that is changing?  
Is there anything else that is changing rapidly?  
Arita-san, you can keep it, I will get it later.  
Yes, Ota-kun.

**Ota:** The number of the dots will increase four pieces at a time.

**Teacher:** Okay. I'll note that point.

...

**(Writing)** (Ota) Increase 4 at a time.

**Teacher:** Increasing by four pieces. Yes.

**Student:** Ota-kun also said that, if the dots, which increased by four pieces, were connected by lines,  
Then it would make a square and the squares would be increasing.

Teacher: Oh, you have said something interesting. You can see the square.  
Write it down please.  
Yoshida is sharp.  
The squares have increased.

(Writing) (Yoshida) The square is increasing.

Teacher: Okay, the squares are increasing rapidly.  
I see.. Yes? ...  
Do you know anything that increases rapidly like this in the world? (Is there anything) in your everyday life?

Student: Debt.

Teacher: What?

Student: I said debt.

Teacher: Debt! You —  
Debt. Is there anything that—"pohn!"—increases rapidly in your daily life?

Students: There's nothing, Kurosawa-sensei!

Teacher: Is there really nothing?

Students: Increase? This is tough. What?

Teacher: Nothing?  
What? Water? Water?

Student: If the water falls. . .

Teacher: I see, the water falls—" pohn"—then it will increase, I see. I see. Water.  
Water. Like water goes "pohn" if stone falls into the water—"pohn."

Teacher: What else? Are there any other images?

Student: A balloon.

Teacher: What? A balloon?

Student: I mean the air makes it bigger.

Teacher: (Blowing) Like this? A balloon, as you blow—"bwah"—into it more and more—"bwah"—is that what you mean?

Teacher: It will expand and in the end—BANG! I see.  
I see. I came up with a much better idea when I saw this.  
I came up with cell division.

Student: No such form exists.

Teacher: You think so? What is it then? The thing which will increase. Bacteria!

Umeki: It can increase in a fixed pattern.

Teacher: Wait a minute. Umeki said a great thing. Say it again.

Umeki: The black dots will increase in a fixed pattern.

Teacher: Since the dots will increase in a fixed pattern, then you want to say it looks like what?

Umeki: A virus.

Teacher: A virus?

Students: ... It is not constant.

Teacher: Isn't a virus constant?

Students: It's not constant.

- Teacher:** Is that so? We never know.  
Natural science behaves by rules, the virus may increase constantly.  
Virus. I understand.  
"Pohn!" There's one (black dot). This is the beginning. This—I wonder how we should do this.  
This is after one second.
- Students:** ...
- Teacher:** It's fast? After one minute.  
Then this diagram is after one minute.  
"After one minute." After one minute.  
So this is the diagram after the second minute. Therefore, after that—one minute later, one minute later—the virus...  
The black dots became a virus. It's like the movie, *My Neighbor Totoro*, (about) that ghost who had a virus.  
Kobayashi, how much will the virus be increased after 5 minutes, if it increases every one and two minutes?  
It is easy to answer for five minutes or six minutes.  
How many minutes would you like to try?
- Students:** One hour. Forty-three minutes. Ten minutes. Thirty-five minutes.
- Student:** An appropriate number is better.
- Teacher:** Appropriate! What is the most appropriate number of minutes?
- Student:** A number that is easy to calculate.
- Teacher:** That is correct. That is right. Umeki.
- Umeki:** There is no number that is hard to calculate...
- Teacher:** There is no number that is hard to calculate.  
Umeki, speak. Umeki please make a problem. Please give us your virus problem. Please, go ahead.
- Umeki:** Well, a virus will ... 48 minutes later.
- (Writing)** (Umeki) A virus after 48 minutes, how many are there?
- Teacher:** Then, from now. Okay? Is there anything about the problem that you don't understand?  
Is it hard for you to calculate 48 minutes? Is it all right?  
Yes, what is it?
- Students:** ...
- Teacher:** Just a moment. Did you try to say the answer? Just a minute. Is this okay as a question? 48 minutes later.  
Then, I will give you 3 or 4 minutes from now, to calculate how much the virus will increase after 48 minutes.  
Start now.

The students worked independently on the problem for about eight minutes. During this time, Kurosawa-sensei walked among the students. He watched them work and occasionally stopped to talk with individuals. When he felt enough students were ready, he called the class together to discuss the problem.

Kurosawa-sensei surveyed the students to find out their answers. Most students answered 193; a few had 189. Kurosawa-sensei began the class discussion, concentrating on the solutions that led to 189 as the answer.

Then, let's start with someone who has the answer 189.

Kawamura-san, please.

**Kawamura:** It increases by 4 pieces 47 times, so that's 47 times 4.

And there is the dot in the center, but that does not increase 4 pieces at a time, so you only need to add 1.

So it is 47 times 4 plus 1.

**Student:** ...

**Teacher:** You can speak after Kawamura-san is done.

**(Writing)** (Kawamura) 47 times, it increases by 4. 47 times 4, plus 1 in the center. 47 times 4 plus 1 is 189.

**Kawamura:** The dots surrounding the four increase 4 at a time,

But the first time is not included, and it increases by 4 pieces 47 times which is 188,

And the dot in the middle is added to it.

Does anyone have an objection to my answer?

**Yoshida:** Kawamura says that four pieces increased 47 times,

This means that since four pieces will increase 47 times after one minute,

Then I think 5, instead of 1, should be added for the first minute.

**(Writing)** (Yoshida) That is what happens one minute later therefore, must add five.

**Yoshida:** Does anybody have any questions regarding my answer?

**Teacher:** Kawamura-san...

**Kawamura:** ...

**Teacher:** You should say your opinion.

Yoshida ...you should name someone...

**Yoshida:** Dobashi-kun.

**Dobashi:** I agree with Yoshida's opinion.

**Teacher:** Ikeda-kun, did you understand?

**Student:** Yes.

**Teacher:** Kawamura-san, are you okay also?

**Kawamura:** Yes.

**Teacher:** It does not become 193 after adding 5.

**Student:** It will. 47 times 4 plus 5.

**Teacher:** I see, I see. I understand. You mean that 47 times 4 plus 5. Is that what you mean?

**Student:** 4 times 48.

**Teacher:** Please speak.

**Student:** It is not 47 times 4 plus 5. Well, it's okay if it's 48 times 4, plus 1.

**(Writing)** (Umeki) 48 times 4 plus 1 is correct

**Teacher:** I see. Then, can anyone explain Nakahara's answer?

Okay, not very many people.

So, Nakai-kun please.

- Nakai:** Since four pieces will increase every minute, so four pieces will increase in one minute, 48 times 4 plus 1.
- (Writing)** (Nakai) Every minute it increases by 4. 4 times 48 plus 1.
- Teacher:** ...
- Nakai:** Does anybody disagree with my opinion?
- Teacher:** Nakai's answer is the same as this one.  
But, it is slightly different.  
Is that okay? 48 times 4 plus 1 and 4 times 48 plus 1.  
Can anybody explain the difference between the two?
- Student:** ...
- Teacher:** Someone has a different method. Wait a minute. There are others.  
Then, please explain it.
- Ogawa:** I did 1 plus 4 times 48 ...
- (Writing)** (Ogawa) Another way to describe it is 1 plus 4 times 48.
- Teacher:** Say it again.
- Ogawa:** 1 plus 4 times 48.
- Teacher:** Oh, this way. What did you say?  
Okay, I will let you explain what the differences and similarities are regarding these three methods...  
It is easier to understand if it is marked as A here, and marked as B here.  
Tatsumi-kun, please explain Umeki-kun's way.
- Tatsumi:** ...
- Teacher:** ...  
A is—  
Please come up ...
- Tatsumi:** You mean I should draw a picture?
- Teacher:** I don't know, please ask.
- Tatsumi:** A is, A is... on the number line... it assumes that 48 is 1, and seeks what is... over 4...
- Teacher:** What you're saying is, 48 times 4?
- Tatsumi:** 4 times 48 means 4 multiplied by 48, but  
what it means is that every 1 minute it increases by 4,  
and it is not increasing by 48 every minute,  
So I think it is better to say 4 times 48.
- Teacher:** So A is better? I mean, B is better?
- Tatsumi:** Between A and B, I thought B was better.  
But between B and C, there is a 1 at the very beginning, and I thought that was even better.
- Teacher:** Then C is better?
- Tatsumi:** Yes
- Teacher:** Okay, Tatsumi-kun says that this one says 48 multiplied 4 times,  
and that one says 4 multiplied 48 times.  
So if it is increasing by 4, then B is the better choice. That is his opinion.  
Ok? Anyone disagree?

- B is better?
- Tatsumi:** 48 is a little big, so...
- Teacher:** OK, let's use this. Let's think of 3 minutes elapsed time. Yes, please.
- Tatsumi:** ... 48 is...so every minute, this part, not there, here... increases by 1, by 4. Here, next to the [center] dot, after the first minute there is one dot, and in 2 minutes there are 2, and after 3 minutes there are 3, but then, over here there's 3 more, and 3 more here, and 3 here. So if you multiply 3 times 4, and then add 1 for the one in the center, you will get the answer.
- Teacher:** How's that?
- Now there are some of you who understand. Do you really understand? So after 48 minutes, there will be 48 more lined up here. Understand? After 1 minute, there's one, after 2 minutes there are 2, and after 3 minutes, 3 and so after 48 minutes there will be 48. Please, go ahead.
- Student:** I think that's okay, but the virus increase takes place uniformly 4 at a time, not 4 here at once, then 4 more there, so...
- Teacher:** Hmm.
- So we can think of the increase this way... or as 4 times 3. This is the difference, right?
- Do you see the difference? As long as you see the difference, that's fine. But judging from the manner of the increase, the opinion is that B is better...or I mean C, because it adds the 1. Now, we're starting to run out of time.
- Hmm. Lastly, is there anyone who can produce a formula that will allow for the calculation of the increase no matter how many minutes have elapsed?
- (Writing)** In order to know the number of the viruses after an unspecified number of moments have elapsed....
- Teacher:** What if you want to figure out the number of viruses after an unspecified number of elapsed minutes?

The class continued for only a few more minutes. Kurosawa-sensei ended class by asking students to consider "other scenarios that increase" for the next time they met.

The sixth-grade lesson was followed by a postlesson discussion with the classroom teacher and the Japanese mathematics educators who were in attendance. Participants from the Congress observed the discussion.