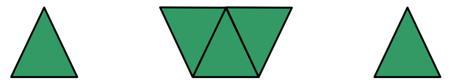
Session Three
Conceptualizing and Representing Linear Relationships
Solution Methods: Polygon Problem

n represents the number of regular polygons
s represents the number of sides of the regular polygon
p represents the perimeter of the composite shape

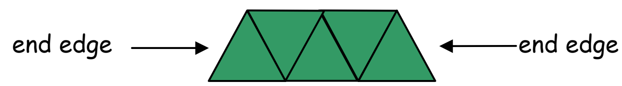
Closed/Explicit/Enumerative Methods



1. Middle and end polygons: $P = (n - 2)(s - 2) + 2(s - 1)$
   $(n - 2)$ is the number of interior polygons
   $(s - 2)$ is the amount contributed to the perimeter by each interior polygon
   $(s - 1)$ is the contribution of each end polygon to the perimeter

The perimeter equals the number of interior polygons times the amount contributed to the perimeter by each interior polygon, plus the contribution toward the perimeter by the two end polygons.

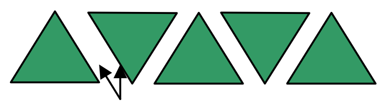
2. Top/bottom/2 end edges: $P = n(s - 2) + 2$
   $(s – 2)$ is the contribution each polygon makes toward the perimeter



The perimeter equals the number of polygons multiplied by the contribution each polygon makes toward the perimeter, plus the two extra end edges. A variation of this is the total number of polygons tops + bottoms + 2: where (tops + bottoms) is $s - 2$ and total number of polygons is n. Again $s - 2$ is the contribution each polygon makes toward the perimeter.

*The notion of "top" + "bottom" and "end" edges is more subtle for other regular polygons with larger sides.

3. Whole - shared sides: $P = ns - 2(n - 1)$
   ns is the number of polygons times the perimeter of each polygon
   $2(n - 1)$ is the number of shared sides, i.e., sides of the polygons not contributing to the perimeter
   P is the total perimeter of each polygon minus the number of the shared sides

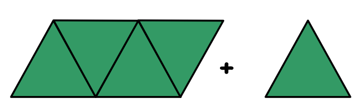


4. Finding an equation from a table
   Using the form $y = mx + b$
   m is the constant change (slope), which is 1 for triangles
   (2 for squares, 3 for pentagons, 4 for hexagons)
   b is the intercept, which is where the number of triangles (polygons) is 0,
   in this case you can find it by subtracting 1 from the perimeter on the first triangle
   This makes no sense within the context, but mathematically allows one to find the equation

Triangles	Perimeter	
0	2	-1
1	3	+1
2	4	+1
3	5	+1
4	6	

Recursive Method

1. Net gain: $P_{(n)} = P_{(n-1)} + (s - 2)$; $P_{\text{initial}} = s$ (at 1 polygon)
   a. Visualizing the geometric relationships:
      Each time we add a new polygon onto our shape,
      the new polygon contributes $s - 1$ sides to the perimeter,
      but also covers one of the old sides.
      The net contribution is $s - 2$ (rate of change/slope).
      Recursive equation: $P(n) = P(n - 1) + (s - 2)$; $P_{\text{initial}} = s$ (at 1 polygon)
      Closed form growing from recursive: $P(n) = (s - 2)(n - 1) + s$
   b. Finding the constant rate of change from a table
      Looking at the perimeter column of the table and noticing
      you add $s - 2$ (1 for triangles, 2 for squares, 3 for pentagons, 4 for hexagons)
      each time to the previous number of faces.



Triangles	Perimeter	
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4	6	