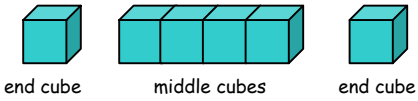
Session Two
Conceptualizing and Representing Linear Relationships
Solution Methods: Cubes in a Line

Closed/Explicit/Enumerative Methods

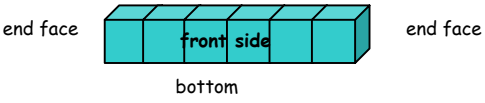
1. Middle and end cubes: $F = (6 - 2)(n - 2) + 2(6 - 1)$ or $F = 4(n - 2) + 2(5)$



Each end cube: 2(5) faces, where each end cube adds 5 faces
The number of middle cubes: (n - 2)
Each middle cube adds 4 faces: $4(n - 2)$
So, the equation looks like: $F = 4(n - 2) + 2(5)$
Where n is the number of cubes and F is the number of unit faces.

# of cubes	Faces
2	(6 - 1) + (6 - 1)
3	(6 - 1) + (6 - 2) + (6 - 1)
10	2(6 - 1) + (10 - 2)(6 - 2)
n	2(6 - 1) + (n - 2)(6 - 2)

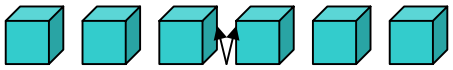
2. Top/bottom/2 side faces + 2 end faces: $F = 4n + 2$ or $F = n4 + 2$



There are two possible ways to see this:
a. The total shape has four sides (top, bottom, 2 side faces) multiplied by the number of cubes (n) plus the two end faces: $4n + 2$
b. Each of the cubes has 4 contributing faces (a top, bottom, and 2 side faces) plus the two end faces: $n4 + 2$

# of cubes	Faces
2	2(4) + 2
3	3(4) + 2
10	10(4) + 2
n	$n4 + 2$

3. Whole – shared sides: $F = 6n - 2(n - 1)$


Shared faces

$6n$ represents six faces per cube
There are pairs of shared faces: $(n - 1)$ number of them, which is $2(n - 1)$
Take the 6 faces per cube and subtract out the shared faces
So, $F = 6n - 2(n - 1)$

# of cubes	Faces
2	6 + 6 - 2
3	6 + 6 + 6 - 2 - 2
10	(10)6 - (10 - 1)2
n	$n6 - (n - 1)2$

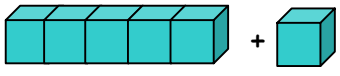
4. Finding an equation from a table

Using the form $y = mx + b$
m is the constant change (slope), which is 4
b is the intercept, which is where the number of cubes is 0,
in this case you can find it by subtracting 4 from the number of face units on the first cube. This makes no sense within the context, but mathematically allows one to find the equation.

# of cubes	Faces
0	2
1	6 (-4)
2	10 (+4)
3	14 (+4)
4	18 (+4)

Recursive Method

1. Net gain: $F(n) = F(n - 1) + 4$; $n_{initial} = 6$



   a. Visualizing the geometric relationships:
   Each time you add a cube, you are adding 5 outside faces but you cover up the existing end face. So, your net gain is (5 - 1) or 4 faces (rate of change/slope).
   Recursive equation: $F(n) = F(n - 1) + 4$; $n_{initial} = 6$
   Closed form growing from recursive: $F(n) = 4(n - 1) + 6$
   b. Finding the constant rate of change from a table. Looking at the faces column of the table and noticing you add 4 each time to the previous number of faces.

# of cubes	Faces
1	6
2	6 + 4
3	10 + 4
4	14 + 4
5	18 + 4