Consecutive Number Exploration

Determining Students' Knowledge of Prerequisite Concepts:

- You are probably already aware of your students' ability to add numbers with sums up to 40, so there is no need to review that during this lesson.
- It is important that your students understand some basic vocabulary before they can be successful in this activity; specifically they need to know
  - ◊ Which numbers are called the *counting numbers* (and that 0 is ***not*** a counting number)
  - ◊ What *consecutive* numbers are.
- Begin a discussion of these concepts by asking your students to write down four or five counting numbers and to write the *smallest* counting number; this way you can quickly walk around the room and determine your students' misconceptions and make sure those become points during your discussion.
- Similarly, ask students to write down examples of *two* consecutive numbers and *three* consecutive numbers.
- Ask your students what it means for numbers to be consecutive; if they say something like, "they come right after each other," ask if they can describe them another way; if needed, you may want to ask a question like, "How are two consecutive numbers related mathematically?"

Focus on children's thinking:
You want your students to be able to explain that "the difference between two consecutive numbers is one" or "if you add one, you get the next consecutive number." [While the connection between "they come right after each other" and "you add one to get the next one" seems obvious to us, it is not always obvious to children. Children may be able to say the numbers in the correct sequence but think of them as any other list like the letters of the alphabet, days of the week, etc. and not recognize the *quantitative relationship* between consecutive numbers. Building this aspect of number sense is an important foundation for much of later mathematics.

Teaching the Lesson

- Explain that you are going to be looking at sums of consecutive counting numbers.
- Give several examples such as $5 + 6 = 11$, $6 + 7 + 8 = 21$, or $1 + 2 + 3 + 4 + 5 = 15$; put these on the chalkboard and leave them visible as a reminder that *any number of consecutive numbers* can be used in the sums—not just a pair of consecutive numbers.
- Give students the **Consecutive Number Exploration** page and have them work individually for about 20 minutes to try to find sums of consecutive numbers which equal the numbers listed. Do <u>not</u> provide any hints such as, "There may be some numbers which have no sums of consecutive numbers equal to them."
- After a reasonable period of individual work, students should work in groups of 2 or 3 to complete their lists.

- Go around the room to get student responses and record all the sums students have for each number. Ask students to supply a "thumbs up" (I agree) or "thumbs down" (I disagree) for each response; any disagreements need to be explored.
- Ask your students what patterns they notice in their answers and allow them about 10 minutes to work individually to discover and write down patterns. After 10 minutes of working individually, the small groups may again compare and discover more patterns.

- Have your students suggest patterns and record them on the chalkboard. Try to get students to word the descriptions of their patterns in mathematical language, for example, instead of saying that "every other number" is the sum of two consecutive numbers, it is better to say "every *odd* number is the sum of two consecutive numbers."

- Next, give students the student sheet **Consecutive Number Exploration – Looking for Patterns** to work on individually.

- Discuss student answers. How did the patterns help find the needed sums of consecutive numbers.

- Working in small groups, students should try to develop explanations for why the patterns work. It is important for students to realize that patterns that occur in numbers and operations are not just *coincidence*—they are direct consequences of the characteristics of the number system. Always thinking about why patterns work builds important number sense in students.

- In grades 5 and 6, it is appropriate to have students think about how to express some of the relationships they have discovered in terms of a variable, say n. For example, we can write three consecutive numbers as n, $n+1$, and $n+2$. This leads to the fact that the sum of three consecutive numbers is

  $$\begin{aligned} & n + (n+1) + (n+2) \\ = \; & n + n + n + 1 + 2 \\ = \; & 3n + 3 \end{aligned}$$

  which explains the pattern that the sum of three consecutive counting numbers is a multiple of 3 and that this pattern starts with 6 [Since 0 is not a counting number, the first number in your set of 3 consecutive numbers (which is n) cannot be 0; therefore, $n = 1$ gives the smallest of these sums].