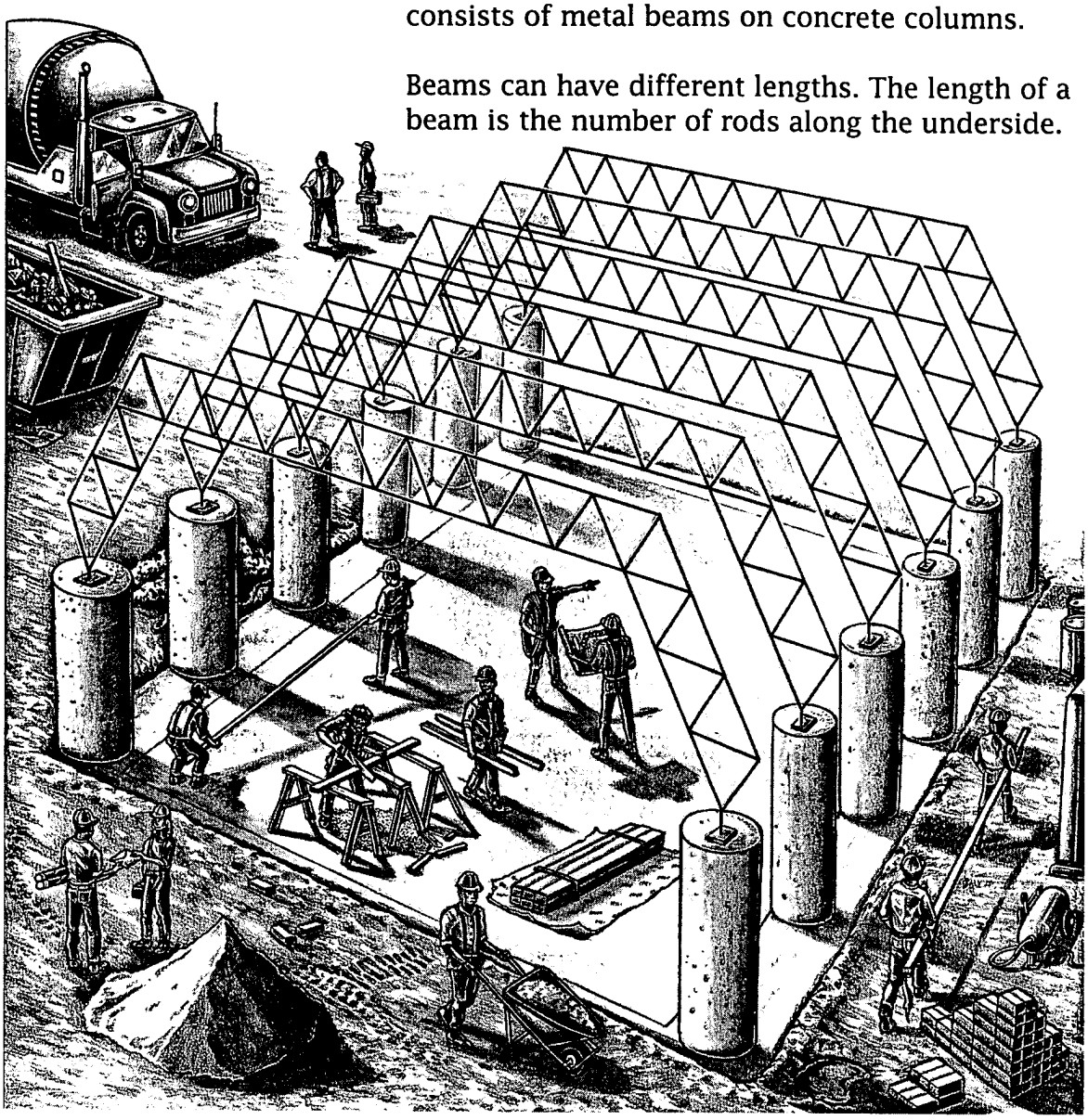
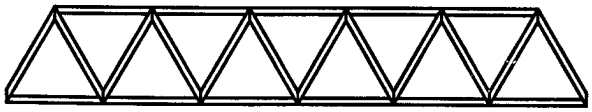
BEAMS

Construction work has begun on a large building that will be used for part of the movie set. The framework consists of metal beams on concrete columns.

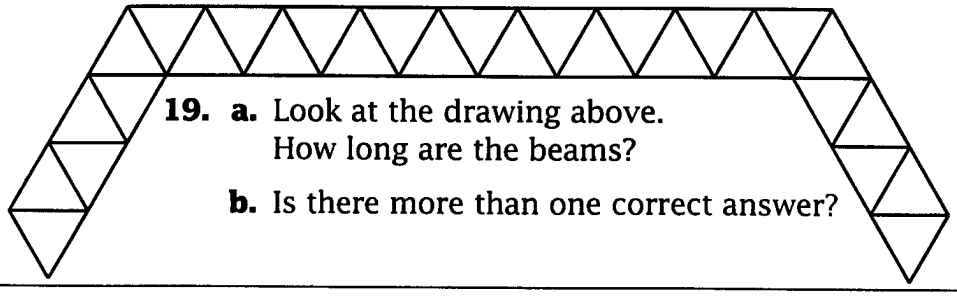
Beams can have different lengths. The length of a beam is the number of rods along the underside.





18. Why is the beam on the left considered to be of length 6?

To make the building for the movie set, three beams are put together as pictured below.

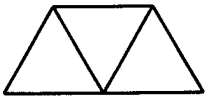
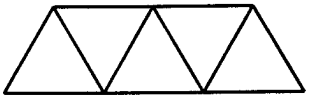
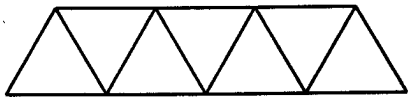


19. a. Look at the drawing above. How long are the beams?

b. Is there more than one correct answer?

In the factory where these beams are made, people want to be able to quickly calculate how many rods are needed for different-length beams.

One way to find the number of rods needed is to look at beams of different lengths.

	Length of Beam	Number of Rods
	1	3
	2	
	3	
	4	

20. **a.** Copy and complete the above table in your notebook.

b. Explain how you found the numbers to fill in the table.

c. Describe some patterns you can see in the table.

d. Add some entries to the table. Make drawings to check your table entries.

Each time the length of the beam increases by one, the same number of rods is added.

21. **a.** How many rods are added each time the beam length increases by one?

b. Write a NEXT-CURRENT formula that describes how the number of rods changes from one row to the next.

c. When might you use this NEXT-CURRENT formula?

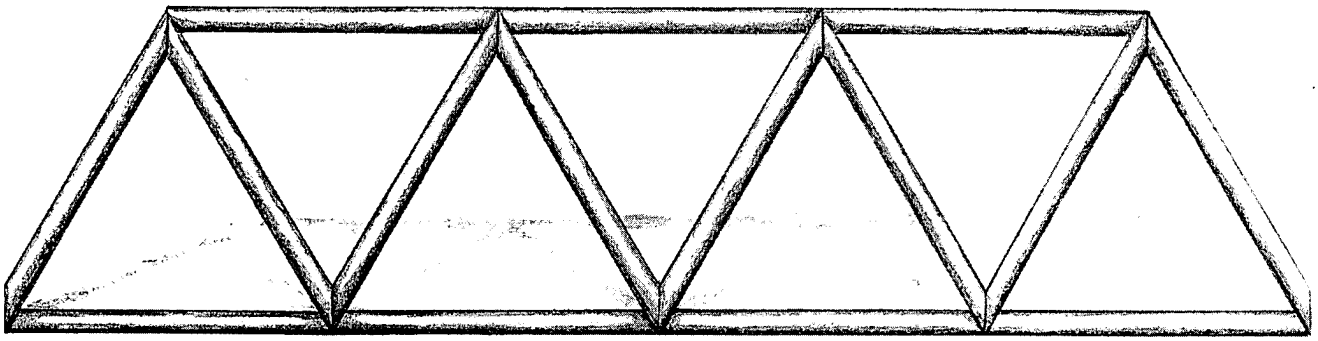
Jim is interested in finding the total number of rods needed for different lengths of beams.

22. **a.** If he wants to find out how many rods are needed for a beam of length 12 and one of length 56, could he use a NEXT-CURRENT formula? Explain.

b. Use toothpicks to build a beam of length 6.

c. Find a direct formula that gives the number of rods (R) needed to build a beam of any length (L).

The people who work for Jim want to help him figure out how many rods are needed for different-length beams. Each of them comes up with a formula that they think will work.



Angelina suggests:
$R = L \times 3 + (L - 1)$

David suggests:
$R = L + (L - 1) + 2L$

Maria suggests:
$R = 3 + (L - 1) \times 4$

Jim is impressed that all of these people want to help, but he wonders whether or not all of these formulas are right.

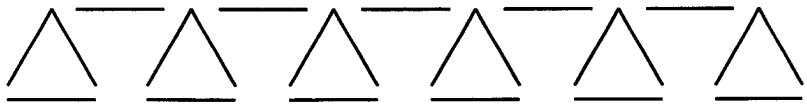
23. a. Check all of the formulas for lengths 7, 15, and 68 to see if they give the same numbers of rods.

b. Are all of the formulas the same? Explain.

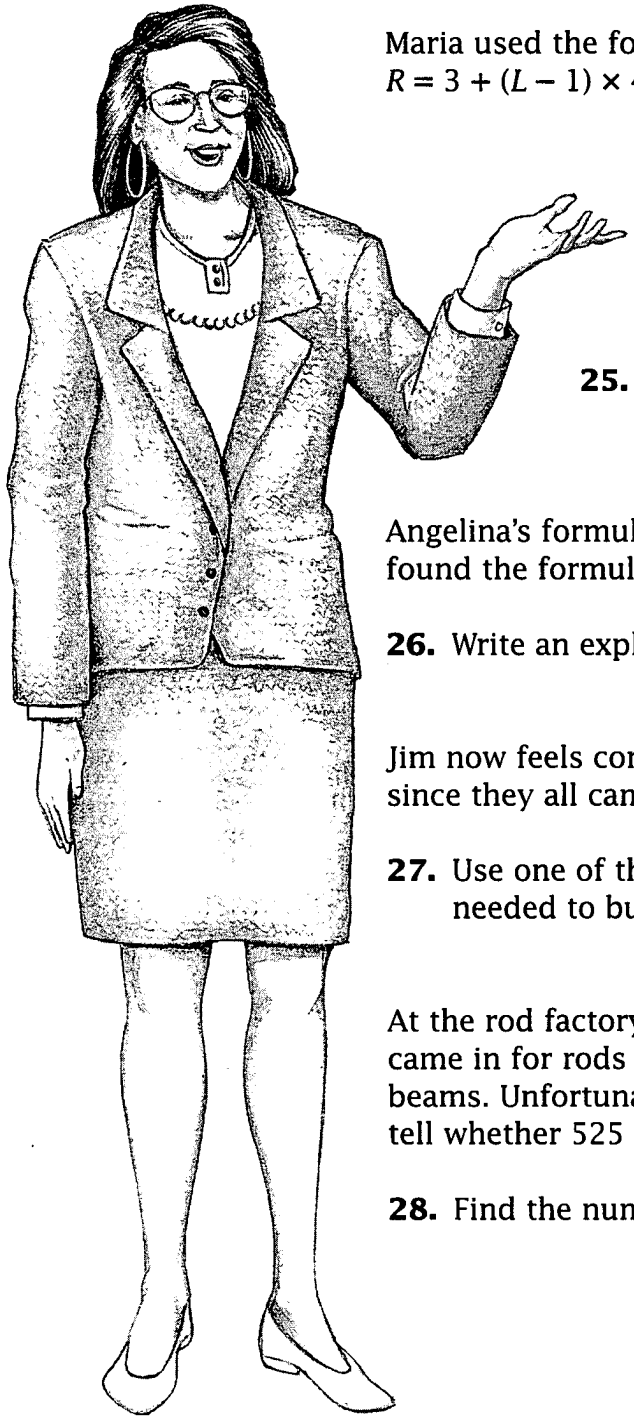
c. Compare these formulas with the one you found for problem **22c.**

Jim is not convinced that all three formulas will give the correct results for all lengths. He decides that if he can figure out where the formulas came from, he will have a better idea about whether or not they will work for all lengths.

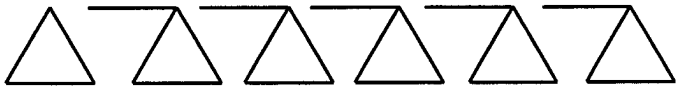
David used the following picture to explain his formula, $R = L + (L - 1) + 2L$:



24. What is a possible explanation for David's formula?



Maria used the following picture to describe her formula, $R = 3 + (L - 1) \times 4$.



25. Finish Maria's explanation. Explain how the above drawings are related to her formula.

Angelina's formula is $R = L \times 3 + (L - 1)$. She tells Jim that she also found the formula by breaking the beam into parts.

26. Write an explanation for Angelina's formula. Use drawings.

Jim now feels confident using any of the formulas. He figures that since they all came from the same picture, they all must work.

27. Use one of the formulas to calculate the number of rods needed to build the movie set shown on page 10.

At the rod factory, many of the orders come in by fax. An order came in for rods to make a building with seven equal-length beams. Unfortunately, the fax was hard to read, and no one could tell whether 525 or 532 rods had been ordered.

28. Find the number that was ordered.

Tile Floor Design

Terry has to design a tile floor for another scene in the movie.

Her design has a gray square in the middle and a white border around it. These floors can be different sizes. Four different sizes are shown on the right.

29. a. Study the design. Find a direct formula to calculate the number of gray tiles (G) needed for any floor number (F).

b. Find a direct formula for the number of white (W) tiles in any floor number.

30. Use drawings to explain that either of the following formulas for the total number of tiles will give the correct result:

$$T = F \times F + 4F + 4$$

or

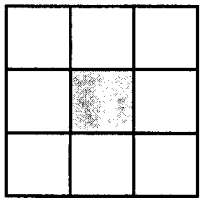
$$T = (F + 2) \times (F + 2)$$

Terry has a total of 144 tiles that she is going to use to build one of these square floors.

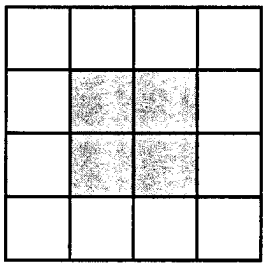
31. a. What floor number is she going to build? Explain.

b. How many gray and how many white tiles does she have? Explain.

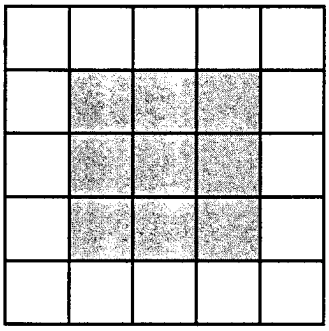
Floor Number 1



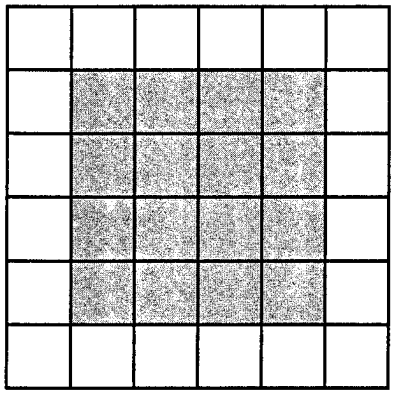
Floor Number 2



Floor Number 3



Floor Number 4



Summary

When solving problems about designs you can:

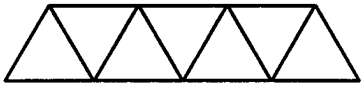
- draw some examples of the design,
- make a table and look for patterns,
- express the patterns as formulas.

You have seen two different types of formulas that can be used to describe a pattern:

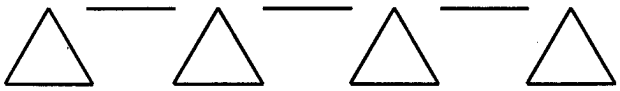
- a NEXT-CURRENT formula, going step–by–step
- a direct formula, working directly with the pattern number

Different direct formulas can be found to describe the same rule or pattern. You can check to see whether or not different formulas give the same result by connecting each formula to the same pattern using a drawing.

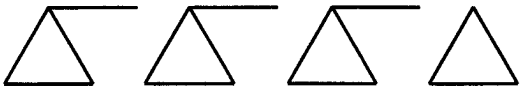
For example, $R = 3L + (L - 1)$ is the same as $R = 4L - 1$ because they both describe the pattern below.



$R = 3L + (L - 1)$ can be connected to the pattern with the following drawing:



$R = 4L - 1$ represents the same pattern, as shown in the following drawing:



Summary, continued

If the meaning is clear, you can leave out the "×" when multiplying. $T = 4 \times L - 1$ is the same as $T = 4L - 1$.

Also, when you are adding or multiplying, you can change the order. $T = (P + 1) \times 3 + 2$ is the same as $T = 3(P + 1) + 2$.

Summary Questions

32. Explain when you would use a NEXT-CURRENT formula and when you would prefer a direct formula.

33. Here are three formulas:

$$T = P + 2 + P$$

$$T = 2P + 2$$

$$T = 2(P + 1)$$

Show that these formulas describe the same pattern.