Building Understanding of Algebraic Representation
The Border Problem, Part 2

TEACHER: What they would do then is, they would say, well, "Is there a way to shorten this?" So, how could we shorten this without doing all the writing and still communicate Joe's method, so that we could understand it? Does anybody have any ideas about how we could do that? You do, Krysta—how?
KRYSTA: By, like, using algebra. Like make, um, the fifteen and fifteen like *x*s and the thirteen and thirteen *y*s and so that you could say, like, *x* plus *x* and then label the square *x* plus *x* and then put all in letters that . . . (*inaudible*).
TEACHER: I really like Krysta's idea of taking some kind of a symbol for, um, like, and she's using a letter. Does anybody know what letters are called in algebra, what they're called? Sharmeen, do you know?
SHARMEEN: Like what they're meant for?
TEACHER: Well, actually, they have a name. The name doesn't really say everything about what they do but . . .
STUDENTS: Variables.
STUDENT: I knew that!
TEACHER: They're called variables. You've heard that word before? OK. So, let's, let's take Krysta's idea and work with it. Let's pick a symbol and let's, let's say, um, for . . . do you want to use *x*, Leo? You guys like *x*? All right. Um, we . . . you could choose *l* for Leo, *a* for Anna, *k* for Krysta; so now what are we going to do?
STUDENTS: *F* for Flores, *h* for Mrs. Humphreys . . .
TEACHER: *H* for Ms. Humphreys. How about, um, well, I have an idea. All right, I have an idea. How about if everyone picks their own variable. So, let's all pick your own variable. I don't care what it is in the book but, let's just, we have to tell what it means. OK, I'm still waiting for quiet and . . . OK, so what we're going to do is we're going to say "Let" and you get to pick your own variable and I'll pick, uh, I don't want to pick one, so, "Let 'something' equal . . . " and this is the important part . . . OK. What I'd like to be able to

do is talk without being interrupted. . . . Up here this first sentence is the most important thing because it says what you need to know, what Joe needs to know to figure out the number of squares. The only thing Joe needs to know is how many unit squares are on one side. That's all he needs to know. So what we want to do is we want to let our symbol equal the number of unit squares on one side. And I guess I will, I'm just going to use x because it seems strange to leave it blank. But you all can choose whatever one you want, but it's really important that you write this part out. "Let x equal the number of unit squares on one side"—and of course you might not have x. Now here's what I'd like you to try to do in your tables. I'd like you to try to translate this into an algebra expression. Like Joe is telling us what to do. Here's a picture of what Joe said to do. Here are some examples of what Joe said, and before we do let's just look at these for a minute. What's staying the same in this arithmetic? Pam, what's staying the same?

PAM: Um, the like, the, you're always adding. You're not, I mean, you could multiply, but you're always adding.

TEACHER: OK. Anything else change, uh change—staying the same? And, uh, what is changing? Pam, I mean, Sarah.

SARAH: Well, the first two numbers are the same numbers and the last two numbers are the same numbers.

TEACHER: OK, so these two are the same, these two are the same, but those are different. So, do you think you could look at this, look at this and look at this and write an algebraic expression? Why don't you try at your table? (*students talk in small groups*).

SHARMEEN: S and so $s + s + (s - 2) + (s - 2)$. Though that's kind of complicated. Is there any other way to put it?

ANTONY: What is it?

SHARMEEN: Um, mine? Was $s + s + (s - 2) + (s - 2)$.

KIM: No we had to, like, um, how about we write a variable for . . . (*inaudible*) . . . make a variable for thirteen.

SHARMEEN: Yeah, oops. Oh, m equals . . . OK, so it's $s + s + m + m$.

TEACHER: (*Addressing whole class*) OK. May I please have your attention at this table for a minute? Do you want to say the theory, your theory, about the other letter?

PAM: OK. Well, um, we knew that we had to add, like, the top and the bottom or up there would be the green and we knew that that would be our, just say our variable is m, so you'd have to add m plus m, but we're trying to figure out a way of, um, subtracting two without saying subtracting two from the sum of m plus m. So we figure that we need, um, another letter for the two sides but we're having a hard time figuring out how to say it.

TEACHER: Why don't you check and see if anybody has any ideas for you.
STUDENT: I don't understand.
TEACHER: Oh, wait a second. Does everyone understand her, the issue at this table?
STUDENTS: No.
TEACHER: Oh. Travis, what's the issue at this table?
TRAVIS: It's, it's uh, . . . I had it . . .
PAM: OK, so we're adding the top and the bottom but we're trying to, uh, figure out how to add the sides but we know we have to subtract two and, we don't know, you can't just say the top plus the bottom minus two because that would mean that you'd be subtracting it from the sum of the top and bottom. So we're trying to figure out, we know we have to use, we think we have to use another letter but we're having a hard time figuring that out; what, like, how to say subtracting two with the other letters.
TEACHER: So since Pam went to all that trouble of explaining, Mindy or Kayla or Joe, would you choose someone in the class to help you out, help give you ideas?
JOE: Stephanie.
STEPHANIE: Um, I think maybe you could, like, do x plus x for the top and the bottom, or whatever letter you're using, and then you could do $(x - 2) + (x - 2)$. And that would give you the, um, the total border.
STUDENT: And then you wouldn't have to do another letter?
STEPHANIE: Yeah, and then you could keep the same letter and you're still taking away two.
PAM: Oh, OK.
TEACHER: Why does that work? Why does that make sense? . . . Melissa, why does that make sense?
MELISSA: All the sides are the same length, so you only need one letter and you have to subtract two.
TEACHER: OK. That whole thing about when, why you might need another letter. Kimberly, you were thinking that you need another letter, right? Why were you thinking that? Because that's a really important thing; when do you need another letter and when don't you? Why did you think you did?
KIMBERLY: Because we have, um, four letters and I was thinking, cause she read her, um, her thing there and I said it was kind of complicated so I was thinking that (*inaudible*) . . .
TEACHER: OK. What do you think now?
KIMBERLY: We don't need it anymore.
TEACHER: You're sure? You're convinced? OK. Um, Travis, were you going to add anything to that? OK. Yeah, Pam.

PAM: Well, the reason I was thinking we needed another letter is because in the beginning we needed two different numbers. So, maybe you needed two different letters.
TEACHER: Oh. Oh, that's . . . yeah, right, right.
PAM: It kind of, like, if you were talking about what's the same and what's different. So in the algebraic formula it's, the things that are the same and different are not the same and different on the numbers.
TEACHER: Right.
PAM: So, I think that when you asked what was the same and what was different, it kind of confused me.
TEACHER: I'm glad it confused you in that way because it brought out a really important thing that we're going to be grappling with in some other problems. Because sometimes you are going to need a different letter, and sometimes you're not. Why don't you need another letter in this case? I mean, like, if you can do it without another letter, you want to keep it simpler. Why can we, why can we? Sarah. Yeah, this Sarah. Sarah Stanley.
SARAH: Because you can do, you can do things to that, uh, letter, to make it the number that you want without using a different letter for that number.
TEACHER: And in this case the thing that you're talking about doing is . . .
SARAH: I said, well, I used *s* as my, um, . . .
TEACHER: OK, so let's write down *s*; I'll write down *s* for Sarah.
SARAH: OK. And, so I said, in parentheses, I said *s* times two.
TEACHER: Now Sarah, I'm going to stop you here because I agree with you that *s* time two is the same as *s* plus *s*, but I want to keep it as consistent with the way Joe is seeing it as possible, so I'm just going to say *s* plus *s*.
SARAH: OK.
TEACHER: OK.
SARAH: And then, like, that's in parentheses, and then I, outside of the parentheses, I put + (s x 2 – 4).
TEACHER: Oh . . .
PAM: But that's a different method.
TEACHER: OK. So maybe that's a different method. Can you think about what Joe would actually do and actually that's another method I want to come back to. That's interesting. So, Sarah, and actually, I'm going to write this down. Sarah wrote, *s* times two, Sarah, and then you wrote, "+ (s x 2 – 4)." We're going to have to come back to that. I don't want to lose it so I'll save this transparency. But what would Joe have done exactly? Travis.
TRAVIS: *S* plus *s* and then plus *s* minus two.
TEACHER: OK.

TRAVIS: And then plus *s* minus two.
TEACHER: Um, let's see. Who would read their sentence to us? I know the bell is about to ring but would anybody . . . oh darn . . . OK. Think about this tonight and tomorrow we'll get to work on it some more. Tonight's homework is . . .