BGSU Mathematics Competition
February 24th 2024 **A** (above Calculus II)

No cell phones are allowed. Show all your work. Justify your answers.

(1) Decide which of the following numbers is larger 2024^{2023} or 2023^{2024}.

(2) The numbers 1 to 50 are written in a row. Can the sign + and − be placed between them such that the value of the resulting expression is 0?

(3) Suppose that either a RED or BLUE or GREEN color is associated with each natural number (assume that all 3 colors are used.) Is it possible that by adding any two natural numbers "of different colors" we get a natural number "of the 3rd color"?

(4) Let f be the one-to-one and onto function

$$f : \{1, 2, \ldots, 6\} \to \{1, 2, \ldots, 6\},$$

given by $f(1) = 2$, $f(2) = 3$, $f(3) = 1$, $f(4) = 5$, $f(5) = 6$, $f(6) = 4$.
(i) Find a one-to-one onto function $g : \{1, 2, \ldots, 6\} \to \{1, 2, \ldots, 6\}$ such that $g \circ g = f$.
(ii) Can you find more than one such function g?
(iii) Determine (with proof) how many such functions g there are.

(5) Let f be a continuous function $f : \mathbb{R} \to \mathbb{R}$. Suppose that $[f(x)]^2 = f(x)$ for every number $x \in \mathbb{R}$. Prove that f must be a constant function.

(6) Prove that no matter how we choose 51 distinct natural numbers from the set

$$\{1, 2, 3, \ldots, 99, 100\}$$

at least two of them must be consecutive.

(7) A bag contains a number of marbles of which 78 are red, 24 are blue, and the rest are green. If the probability of selecting a green marble is 1/3, what is the probability of selecting a red marble?

(8) A spider has one sock and one shoe for each of its eight legs. In how many different **orders** can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

Registration 2024 BGSU Mathematics Competition;

Your NAME:

E-mail:

(Optional)

Math class you are taking this semester/year:

Name of your instructor(s):

---

1)

2)

3)

4)

5)

6)

7)

8)

Total: