

SAS output for Hierarchical Linear Models

Unconditional Random Intercept Model

$$\text{MATHACH}_{ij} = \gamma_{00} + u_{0j} + r_{ij}$$

The Mixed Procedure

Convergence criteria met.

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate | Standard Error | Z Value | Pr > Z |
|-----------|---------|----------|----------------|---------|--------|
| Intercept | SCHOOL | 8.6097 | 1.0778 | 7.99 | <.0001 |
| Residual | | 39.1487 | 0.6607 | 59.26 | <.0001 |

Fit Statistics

| | |
|---------------------------------|---------|
| -2 Res Log Likelihood | 47116.8 |
| AIC (smaller is better) | 47120.8 |
| AICC (smaller is better) | 47120.8 |
| BIC (smaller is better) | 47126.9 |

Solution for Fixed Effects

| Effect | Estimate | Standard Error | DF | t Value | Pr > t |
|-----------|----------|----------------|-----|---------|---------|
| Intercept | 12.6370 | 0.2443 | 159 | 51.72 | <.0001 |

Random Intercept Model with a level 2 predictor

$$\text{MATHACH}_{ij} = \gamma_{00} + \gamma_{01}(\text{MEANSES}) + u_{0j} + r_{ij}$$

Convergence criteria met.

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate | Standard Error | Z Value | Pr > Z |
|-----------|---------|----------|----------------|---------|--------|
| Intercept | SCHOOL | 2.6357 | 0.4036 | 6.53 | <.0001 |
| Residual | | 39.1578 | 0.6608 | 59.26 | <.0001 |

Fit Statistics

| | |
|--------------------------|---------|
| -2 Res Log Likelihood | 46961.3 |
| AIC (smaller is better) | 46965.3 |
| AICC (smaller is better) | 46965.3 |
| BIC (smaller is better) | 46971.4 |

Solution for Fixed Effects

| Effect | Estimate | Standard Error | DF | t Value | Pr > t |
|-----------|----------|----------------|-----|---------|---------|
| Intercept | 12.6495 | 0.1492 | 158 | 84.77 | <.0001 |
| MEANSES | 5.8635 | 0.3613 | 158 | 16.23 | <.0001 |

Type 3 Tests of Fixed Effects

| Effect | Num DF | Den DF | F Value | Pr > F |
|---------|--------|--------|---------|--------|
| MEANSES | 1 | 158 | 263.37 | <.0001 |

Random-coefficient Model with a Level 1 predictor

The Mixed Procedure

$$\text{MATHACH}_{ij} = \gamma_{00} + \gamma_{10}(\text{SES} - \text{MEANSES}) + u_{0j} + u_{1j}(\text{SES} - \text{MEANSES}) + r_{ij}$$

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate | Standard Error | Z Value | Pr > Z |
|----------|---------|----------|----------------|---------|---------|
| UN(1,1) | SCHOOL | 8.6769 | 1.0786 | 8.04 | <.0001 |
| UN(2,1) | SCHOOL | 0.05075 | 0.4062 | 0.12 | 0.9006 |
| UN(2,2) | SCHOOL | 0.6940 | 0.2808 | 2.47 | 0.0067 |
| Residual | | 36.7006 | 0.6258 | 58.65 | <.0001 |

Fit Statistics

| | |
|--------------------------|---------|
| -2 Res Log Likelihood | 46714.2 |
| AIC (smaller is better) | 46722.2 |
| AICC (smaller is better) | 46722.2 |
| BIC (smaller is better) | 46734.5 |

Null Model Likelihood Ratio Test

| DF | Chi-Square | Pr > ChiSq |
|----|------------|------------|
| 3 | 1065.70 | <.0001 |

Solution for Fixed Effects

| Effect | Estimate | Standard Error | DF | t Value | Pr > t |
|-----------|----------|----------------|------|---------|---------|
| Intercept | 12.6493 | 0.2445 | 159 | 51.75 | <.0001 |
| cses | 2.1932 | 0.1283 | 7024 | 17.10 | <.0001 |

Random-coefficient Model with predictors from two different levels

$$\text{MATHACH}_{ij} = \gamma_{00} + \gamma_{01}(\text{MEANSES}) + \gamma_{02}(\text{SECTOR}) + \gamma_{10} (\text{SES} - \text{MEANSES}) + \gamma_{11}(\text{MEANSES})^* (\text{SES} - \text{MEANSES}) + \gamma_{12}(\text{SECTOR})^* (\text{SES} - \text{MEANSES}) + u_{0j} + u_{1j}(\text{SES} - \text{MEANSES}) + r_{ij}$$

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate | Standard Error | Z Value | Pr > Z |
|----------|---------|----------|----------------|---------|--------|
| UN(1,1) | SCHOOL | 2.3817 | 0.3717 | 6.41 | <.0001 |
| UN(2,1) | SCHOOL | 0.1926 | 0.2045 | 0.94 | 0.3464 |
| UN(2,2) | SCHOOL | 0.1014 | 0.2138 | 0.47 | 0.3177 |
| Residual | | 36.7212 | 0.6261 | 58.65 | <.0001 |

Fit Statistics

| | |
|--------------------------|---------|
| -2 Res Log Likelihood | 46503.7 |
| AIC (smaller is better) | 46511.7 |
| AICC (smaller is better) | 46511.7 |
| BIC (smaller is better) | 46524.0 |

Null Model Likelihood Ratio Test

| DF | Chi-Square | Pr > ChiSq |
|----|------------|------------|
| 3 | 220.57 | <.0001 |

Solution for Fixed Effects

| Effect | Estimate | Standard Error | DF | t Value | Pr > t |
|--------------|----------|----------------|------|---------|---------|
| Intercept | 12.1136 | 0.1988 | 157 | 60.93 | <.0001 |
| MEANSES | 5.3391 | 0.3693 | 157 | 14.46 | <.0001 |
| SECTOR | 1.2167 | 0.3064 | 157 | 3.97 | 0.0001 |
| cses | 2.9388 | 0.1551 | 7022 | 18.95 | <.0001 |
| MEANSES*cses | 1.0389 | 0.2989 | 7022 | 3.48 | 0.0005 |
| SECTOR*cses | -1.6426 | 0.2398 | 7022 | -6.85 | <.0001 |

Unconditional Growth Curve Model without predictors

$$Y_{ij} = [b_{00} + b_{10} \text{TIME}_{ij}] + [u_{0j} + u_{1j} \text{TIME}_{ij} + r_{ij}]$$

Convergence criteria met.

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate | Standard Error | Z Value | Pr > Z |
|----------|---------|----------|----------------|---------|---------|
| UN(1,1) | id | 1198.78 | 318.38 | 3.77 | <.0001 |
| UN(2,1) | id | -179.26 | 88.9634 | -2.01 | 0.0439 |
| UN(2,2) | id | 132.40 | 40.2107 | 3.29 | 0.0005 |
| Residual | | 159.48 | 26.9566 | 5.92 | <.0001 |

Fit Statistics

| | |
|---------------------------------|--------|
| -2 Res Log Likelihood | 1266.8 |
| AIC (smaller is better) | 1274.8 |
| AICC (smaller is better) | 1275.1 |
| BIC (smaller is better) | 1281.0 |

Null Model Likelihood Ratio Test

| DF | Chi-Square | Pr > ChiSq |
|----|------------|------------|
| 3 | 120.90 | <.0001 |

Solution for Fixed Effects

| Effect | Estimate | Standard Error | DF | t Value | Pr > t |
|------------------|----------|----------------|-----|---------|---------|
| Intercept | 164.37 | 6.1188 | 34 | 26.86 | <.0001 |
| time | 26.9600 | 2.1666 | 104 | 12.44 | <.0001 |

Growth Curve Model with a level 1 predictor

The Mixed Procedure

$$Y_{ij} = b_{00} + b_{10}(\text{TIME})_{ij} + b_{01}(\text{COVAR})_{ij} + b_{11}(\text{COVAR})(\text{TIME})_{ij} + u_{0j} + u_{1j}(\text{TIME})_{ij} + r_{ij}$$

Covariance Parameter Estimates

| Cov Parm | Subject | Estimate | Standard Error | Z Value | Pr > Z |
|----------|---------|----------|----------------|---------|--------|
| UN(1,1) | id | 1236.41 | 332.40 | 3.72 | <.0001 |
| UN(2,1) | id | -178.23 | 85.4298 | -2.09 | 0.0370 |
| UN(2,2) | id | 107.25 | 34.6767 | 3.09 | 0.0010 |
| Residual | | 159.48 | 26.9566 | 5.92 | <.0001 |

Fit Statistics

| | |
|--------------------------|--------|
| -2 Res Log Likelihood | 1260.3 |
| AIC (smaller is better) | 1268.3 |
| AICC (smaller is better) | 1268.6 |
| BIC (smaller is better) | 1274.5 |

Null Model Likelihood Ratio Test

| DF | Chi-Square | Pr > ChiSq |
|----|------------|------------|
| 3 | 120.72 | <.0001 |

Solution for Fixed Effects

| Effect | Estimate | Standard Error | DF | t Value | Pr > t |
|-------------|----------|----------------|-----|---------|---------|
| Intercept | 164.37 | 6.2061 | 33 | 26.49 | <.0001 |
| time | 26.9600 | 1.9939 | 103 | 13.52 | <.0001 |
| ccovar | -0.1136 | 0.5040 | 33 | -0.23 | 0.8231 |
| time*ccovar | 0.4329 | 0.1619 | 103 | 2.67 | 0.0087 |