Introduction to Structural Equation Modeling

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CFDR Workshop Series

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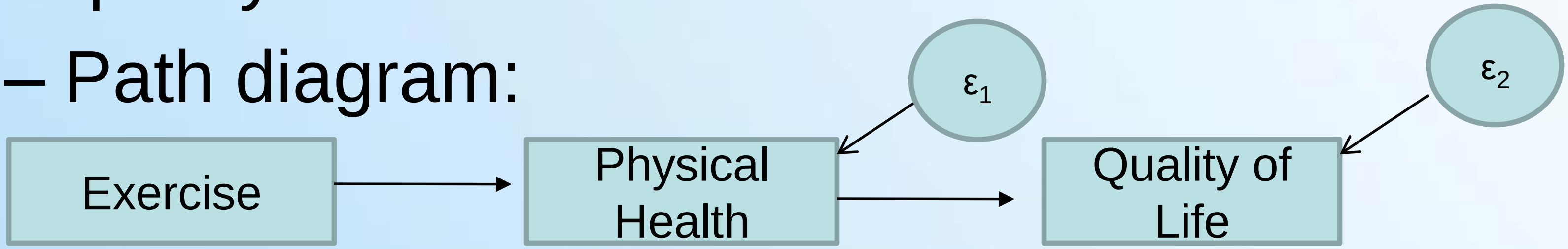
Outline of Presentation

- Basic concepts of structural equation model (SEM)
- What are advantages of SEM over OLS?
- Steps of fitting SEM
- An example of fitting SEM
- Different types of SEM
- Strengths and Limitations of SEM
- Conclusions

Basic Concepts of SEM

- Link conceptual models, path diagrams, and mathematic equations together:
  - Conceptual model: More exercise leads to better physical health, which then increases quality of life
  - Path diagram:



  - Equations:
    - Physical Health= $\mu_1 + \beta_1 * \text{Exercise} + \varepsilon_1$
    - Quality of Life = $\mu_2 + \beta_2 * \text{Physical Health} + \varepsilon_2$

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Jargon of SEM

- Variables in SEM
  - Measured variable
  - Latent variable
  - Exogenous variable
  - Endogenous variable
  - Error
  - Disturbance

Relation between Two Variables

- A path with a single headed arrow
  - one variable predicts the other variable
  - one variable is the indicator of the other variable
- A path with a double-headed arrow means that two variables are correlated with each other
- No path means no direct relation between two variables

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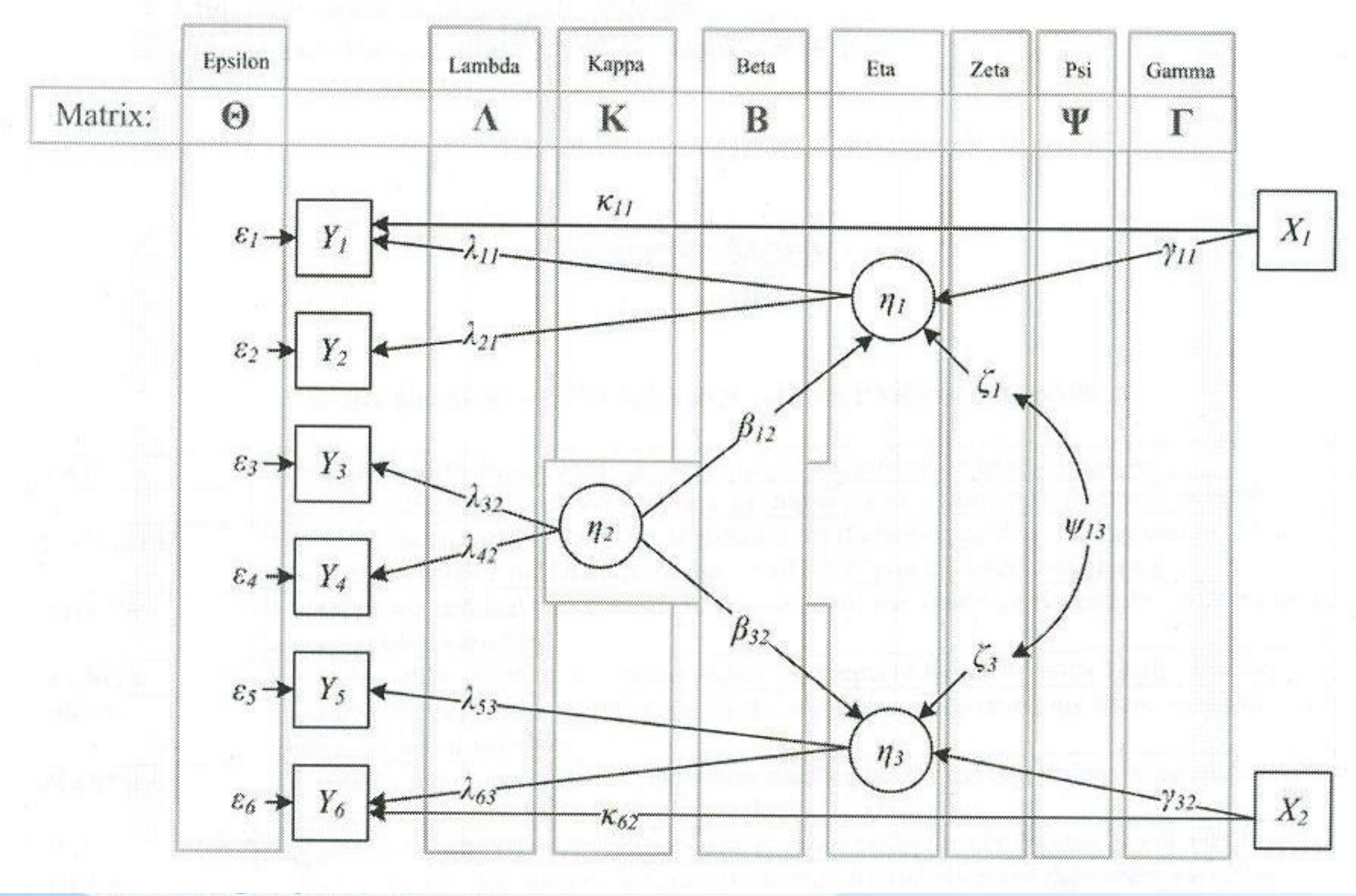
Parameters in SEM



Effects of One Variable on Another Variable

- Direct effect
- Indirect effect
- Total effect

Advantages of SEM over OLS

- Control for measurement errors in observed independent variables, dependent variables, or both.
- Analyze more than one dependent variables at a time
- Distinguish among direct, indirect, and total effects of variables
- Model how Xs influence Ys via other variables
- Test more complex models on three or more waves of longitudinal data

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Steps of Conducting SEM Analysis

- Develop a theoretically based model
- Construct the SEM diagram
- Convert the SEM diagram into a set of structural equations
- Clean data and decide the input data type
- Determine the estimation method
- Run the model and evaluate goodness-of-fit of the model
- Modify the model
- Compare two models and decide if additional modification is needed

Input Data Type

- Raw data
- Correlation matrix
- Covariance matrix
- Covariance matrix and means
- Correlation matrix and standard deviations
- Correlation matrix, standard deviations, and means

Estimation Methods

- ML: Maximum likelihood estimation
- ULS: unweighted least squares estimation
- GLS: generalized least squares estimation

Maximum Likelihood Estimation

- Assume multivariate normality of observed variables
- Is commonly used with large sample size
- Parameter estimates are consistent, asymptotically unbiased, and efficient
- Estimates are normally distributed, which allows for testing statistical significance of parameters
- ML estimates are scale-free

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Unweighted Least Squares Estimation

- Statistically consistent parameter estimates
- No distributional assumption for variables
- Possibly compute tests of significance for model parameter
- Item parameter estimates and fit index are scale dependent
- Parameter estimates are not asymptotically efficient
- No overall test of fit

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Generalized Least Squares Estimation

- Parameter estimates are consistent, asymptotically unbiased, and efficient.
- Estimates are asymptotically normally distributed.
- Like ML, GLS estimates are also scale free.
- Use χ^2 test for model fit

Criteria for Goodness-of-fit of the model

- Overall model fit
  - Chi-Square test (p-value greater than .05)
- Incremental fit indices
  - Comparative Fit Index (CFI >= .90)
  - Non-Normed Fit Index (NNFI >=.90)
- Residual-based Indices
  - Root Mean Square Error of Approximation (RMSEA ,=.05)
  - Standardized Root Mean Square Residual (SRMR <= .05)
  - Root Mean Square Residual (RMR <= .05)
  - Goodness of Fit Index (GFI >= .95)
  - Adjusted Goodness of Fit Index (AGFI >= .90)
- Model Comparison Indices
  - Chi-Square Difference Test
  - Akaike (AIC)
  - Bayesian Information Criterion (BIC)

Modify the Model

- Increase the overall fit of the model
  - Constrain some parameters to be 0
  - Set equal constrains for some parameters
  - Add new paths among variables
- Expected outcome
  - Good overall fit of the model
  - The value of each estimated parameter is significantly different from 0.

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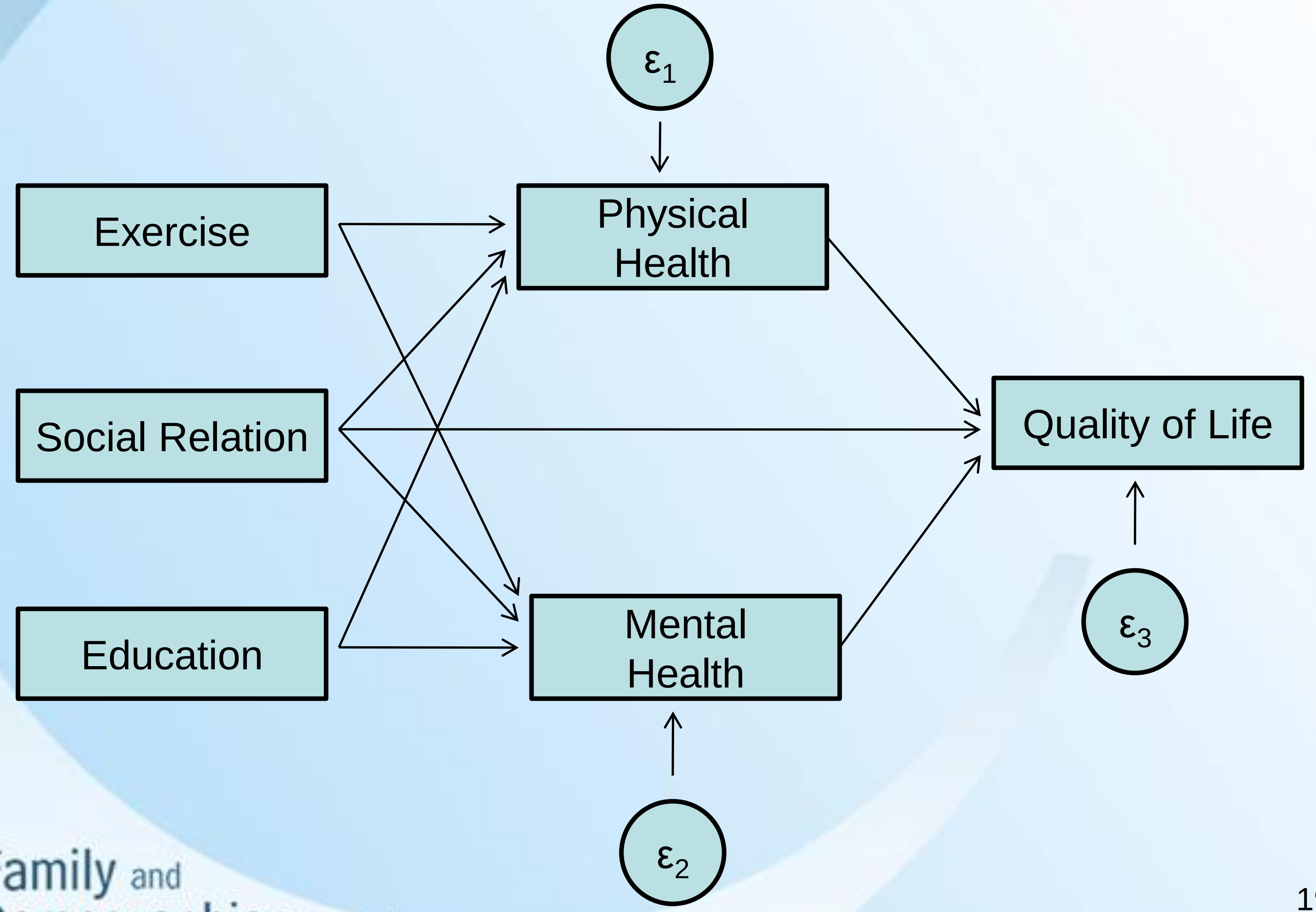
Comparison between Two Models

- Nested models
  - Likelihood ratio test
- Nonnested model
  - Akaike (AIC)
  - Bayesian (BIC)

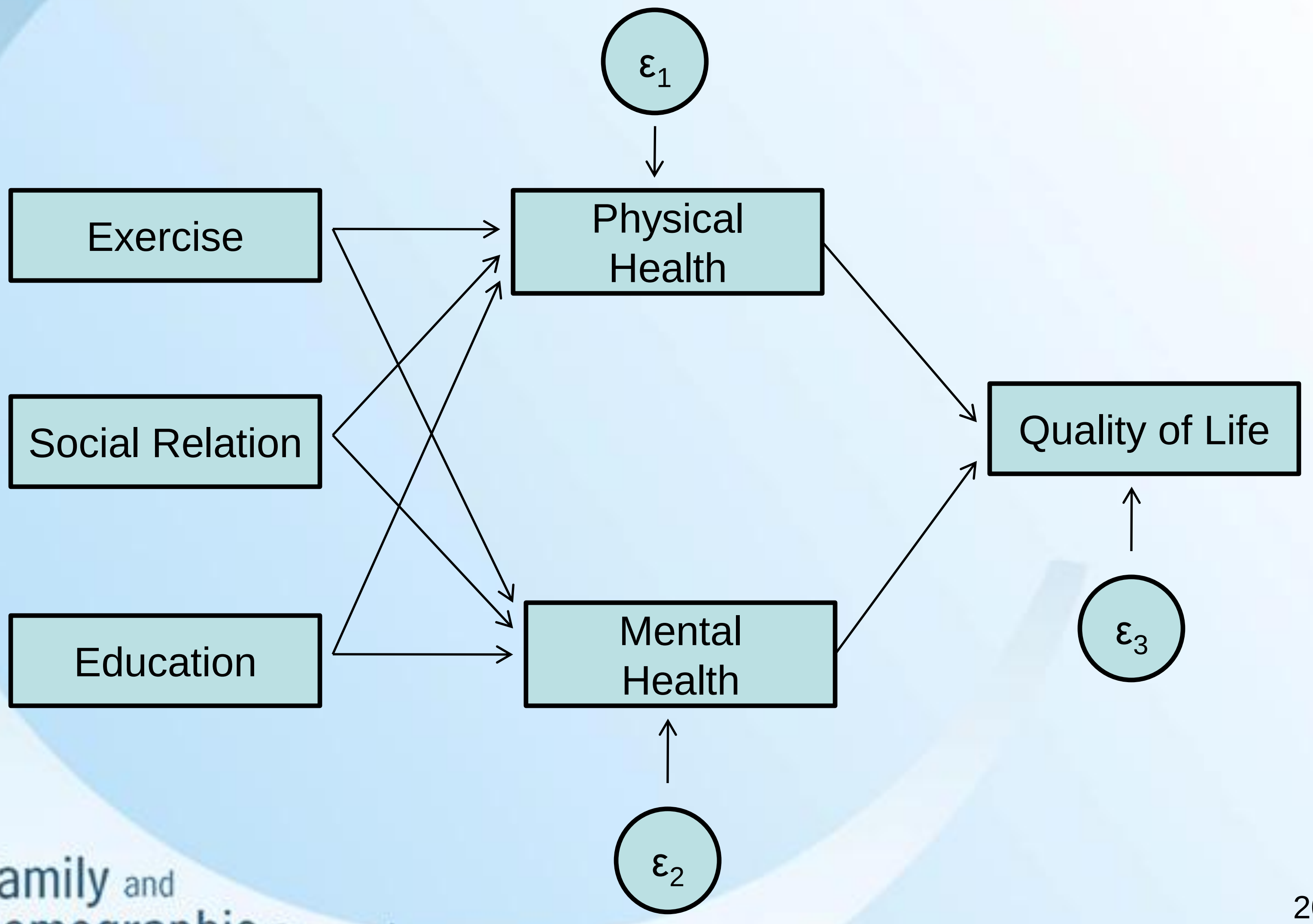
An Example of SEM

- Exercise increases physical health and mental health
- Social relation improves physical health and mental health
- Education enhances physical health and mental health
- Physical health and mental health influence quality of life
- Social relations may or may not have an direct impact on quality of life (hypothesis)

Path Diagram A



Path Diagram B



Goodness-of-Fit for Diagram A

- Chi-Square test: X^2 = 0.757, DF =3, P=.8598
- CFI = 1.000
- RMSEA = 0
- SRMR = 0.001
- Akaike (AIC) = 9143.105
- Bayesian (BIC) = 9206.324

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Result of Path Diagram A

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Y1	ON				
	X1	0.992	0.043	22.979	0
	X2	2.001	0.045	44.618	0
	X3	3.052	0.045	68.274	0
Y2	ON				
	X1	2.935	0.05	59.002	0
	X2	1.992	0.052	38.556	0
	X3	1.023	0.051	19.869	0
Y3	ON				
	Y1	0.507	0.02	25.491	0
	Y2	0.746	0.02	37.914	0
	X2	1.046	0.072	14.54	0
Intercepts					
	Y1	-1.064	0.046	-23.059	0
	Y2	-0.042	0.053	-0.784	0.433
	Y3	1.068	0.063	17.093	0
Residual Variances					
	Y1	1.061	0.067	15.811	0
	Y2	1.408	0.089	15.811	0
	Y3	1.717	0.109	15.811	0

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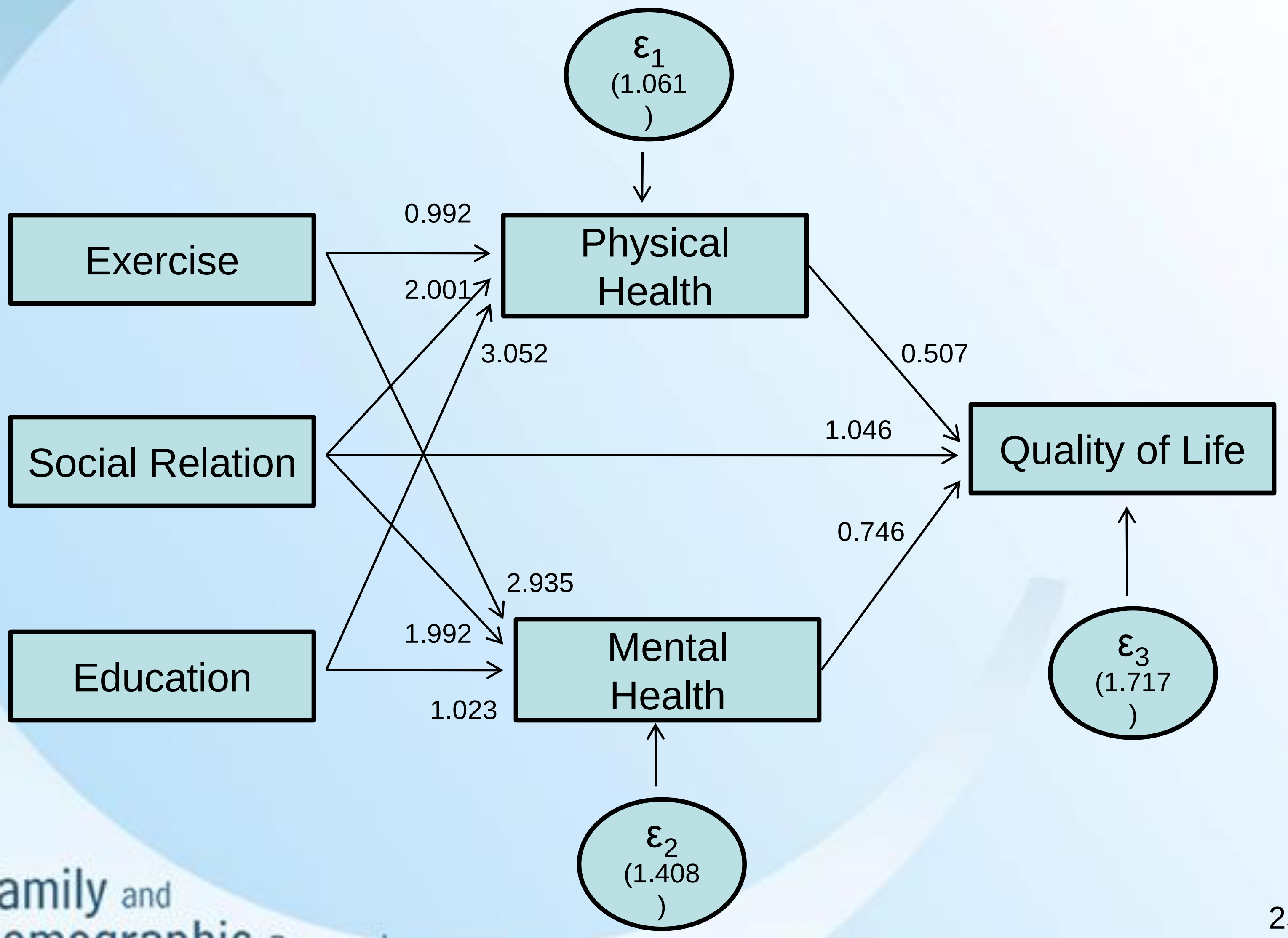
Goodness-of-Fit for Diagram B

- Chi-Square test: X^2 = 177.068, DF =4, P=.0000
- CFI = 0.958
- RMSEA = 0.294
- SRMR = 0.027
- Akaike (AIC) = 9713.416
- Bayesian (BIC) = 9376.420

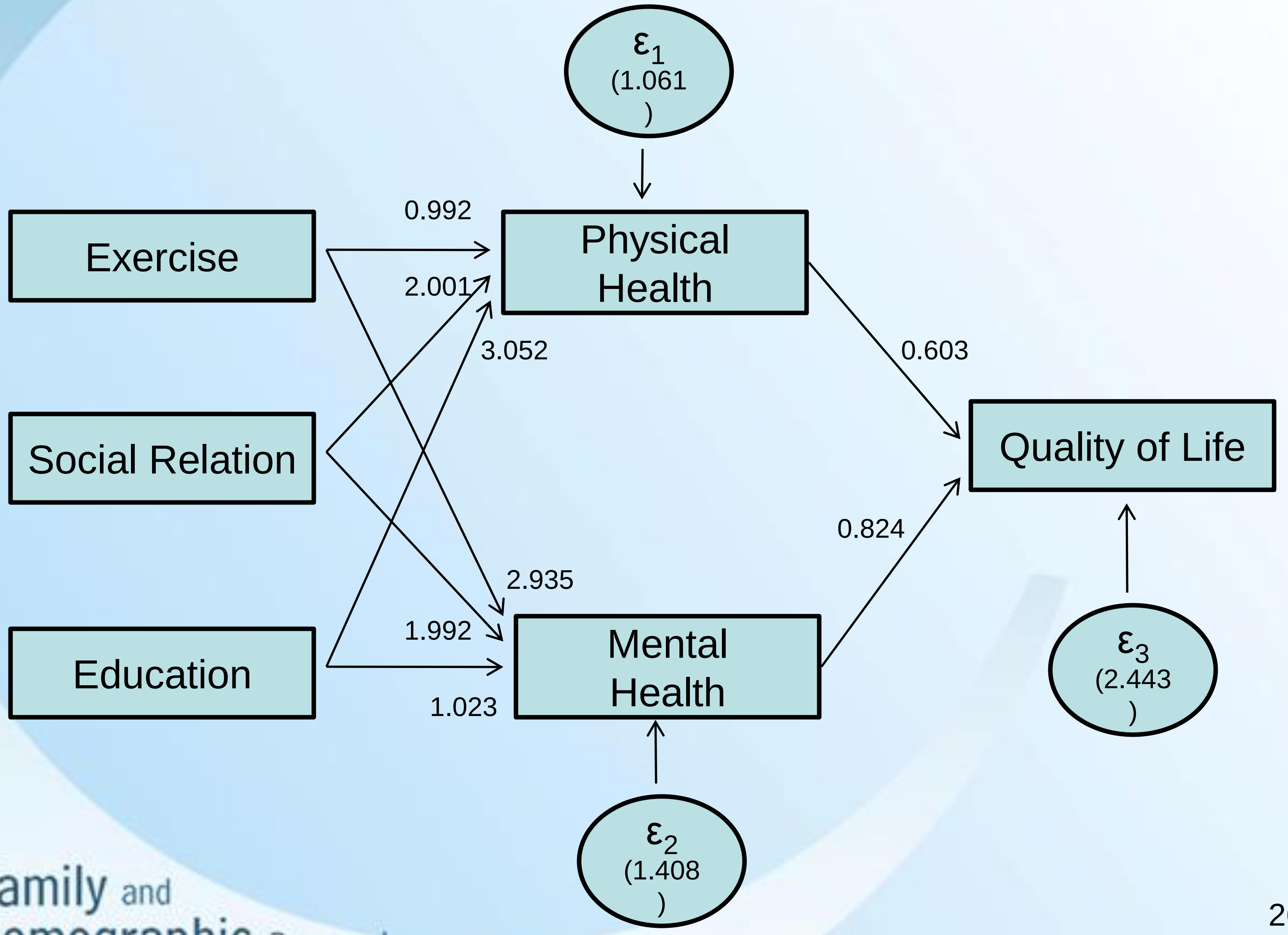
Result of Path Diagram B

					Two-Tailed
		Estimate	S.E.	Est./S.E.	P-Value
Y1	ON				
	X1	0.992	0.043	22.98	0
	X2	2.001	0.045	44.62	0
	X3	3.052	0.045	68.27	0
Y2	ON				
	X1	2.935	0.05	59	0
	X2	1.992	0.052	38.56	0
	X3	1.023	0.051	19.87	0
Y3	ON				
	Y1	0.603	0.022	26.98	0
	Y2	0.824	0.023	36.52	0
Intercepts					
	Y1	-1.06	0.046	-23.1	0
	Y2	-0.04	0.053	-0.78	0.433
	Y3	1.145	0.074	15.41	0
Residual Variances	Y1	1.061	0.067	15.81	0
	Y2	1.408	0.089	15.81	0
	Y3	2.443	0.155	15.81	0

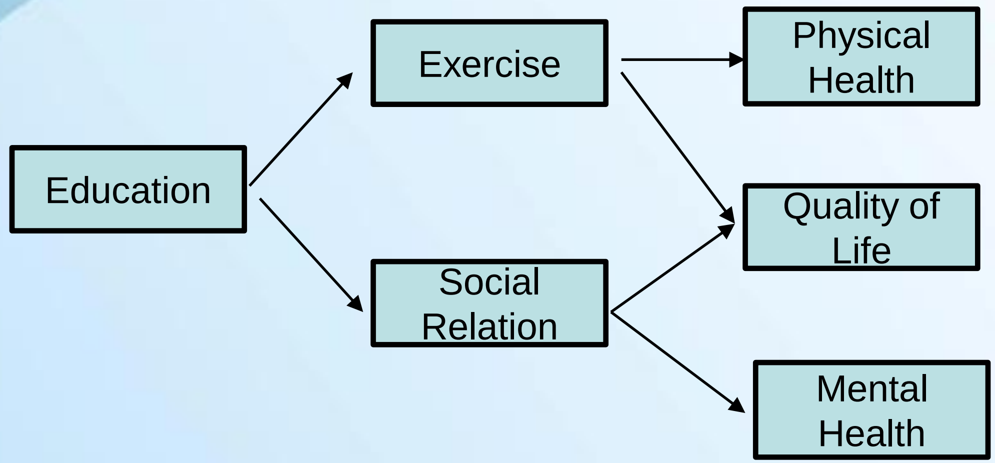
Results for Path Diagram A



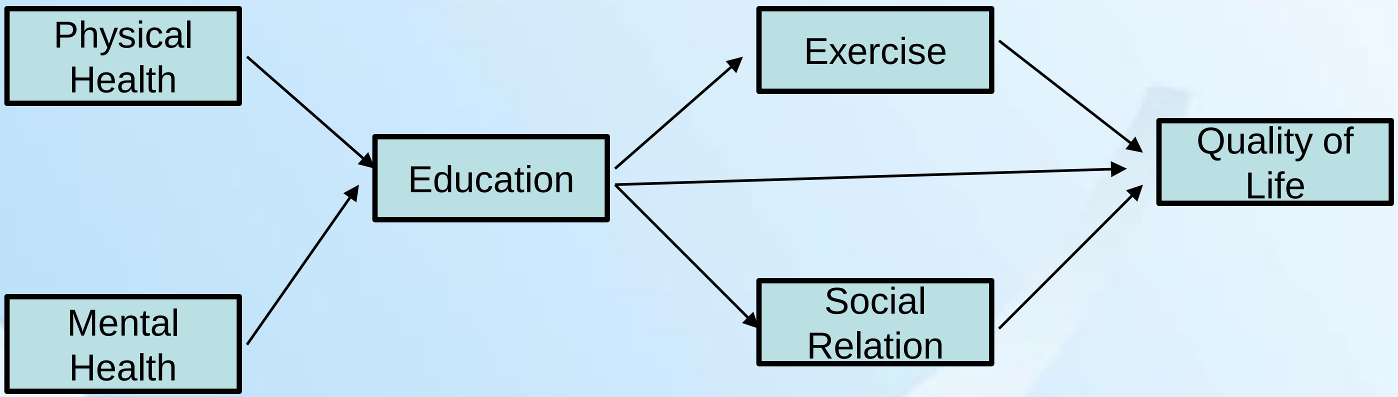
Results for Path Diagram B



Alternative models



Alternative Model 1



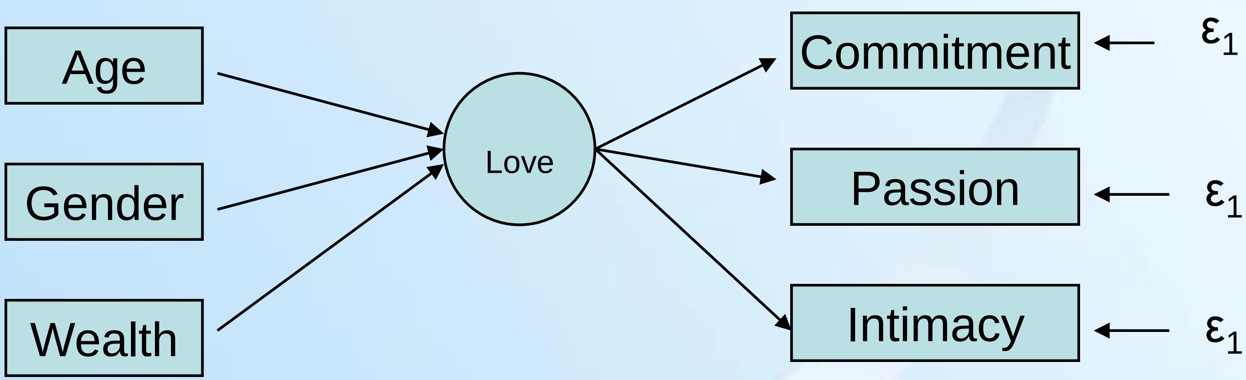
Alternative Model 2

Different Types of SEM

- Path model
- Auto-regressive model
- Growth curve model
- Hierarchical linear model
- Mixture model
- Latent class analysis

Different Types of SEM (Cont.)

- Factor analysis models
  - Confirmatory factor analysis
  - Second-order factor models
- Full structural equation models
  - Mimic model



A Few SEM Applications in JMF

- Schoppe-Sullivan, Sarah J, Alice C. Schermerhorn, and E. Mark Cummings. 2007. "Marital Conflict and Children's Adjustment: Evaluation of the Parenting Process Model." *Journal of Marriage and Family* 69: 1118-1134.

- Vandewater, Elizabeth A. and Jennifer E. Lansford. 2005. "A Family Process Model of Problem Behaviors in Adolescents." *Journal of Marriage and Family* 67: 100-109.

- Mistry, Rashmita S., Edward D. Lowe, Aprile D. Benner, and Nina Chien. 2008. "Expanding the Family Economic Stress Model: Insights from a Mixed-Methods Approach." *Journal of Marriage and Family* 70: 196-209.

An Example of LISREL Codes

LISREL codes for Schoppe-Sullivan, Schermerhorn, and Cummings (JMF 2007, Figure 1)

```
DA NI=19 NO=283 MA=CM
LA FI=data.txt
KM FI=data.txt
SD FI=data.txt
SE
7 8 9 10 11 12 13 14 15 16 1 2 3 4 5 6/
MO NY=10 NX=6 NE=4 NK=1 LY = FI BE=SD PS=DI TE=SY
 LE
 PB-CON PP-CON P-WARM C-SYM
 LK
 M-CONFLI
FI BE 2 1 BE 3 1 BE 3 2
VA 1 LX 1 1 LY 1 1 LY 4 2 LY 6 3 LY 9 4
FR LX 2 1 LX 3 1 LX 4 1 LX 5 1 LX 6 1 LY 2 1 LY 3 1 LY 5 2 LY 7 3 LY 8 3 LY 10 4
PD
OU MI
```

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Strengths of SEM

- Specify various models for different relations among variables, depending on theoretical frameworks
- Distinguish among direct, indirect, and total effect of variables
- Analyze the relations among variables controlling for measurement errors
- Comprehensive statistical tests for identifying and comparing different structural models

Limitations of SEM

- SEM does not establish causal orders among variables if the temporal order of these variables is unknown.

- Missing data and outliers influence the covariance and correlation matrices analyzed.

Limitations of SEM (Cont.)

- A large sample size produces stable estimates of the covariance or correlation among variables, but it make the model easier to be rejected.
- There may be multiple equivalent models that fit data equally well.
- The number of parameters to be estimated cannot exceed the number of known values.

Conclusions

- SEM is a useful analytic technique in situations when independent variables, dependent variables, or both contain measurement errors.
- Even when your variables do not contain measurement errors, SEM allows for better testing theoretical links (i.e., paths) among variables.
- Available software: SAS, LISREL, Amos, EQS, and Mplus.
  - SAS is available on all computers in Williams Hall.
  - LISREL is available in Hayes 025 Lab and Olscamp 207 Lab.
  - Amos, EQS, and Mplus not supported by BGSU

Conclusions (Cont.)

- More readings about SEM:
  - Bollen (1989, Structural Equation Modeling)
  - Kline (1998, Principles and Practice of Structural Equation Modeling)
  - Kaplan (2000, Structural equation Modeling)
  - Raykov & Marcoulides (2000, A First Course in Structural Equation Modeling)

- If you encounter problems running SEM models, feel free to contact me (Hsueh-Sheng Wu, wuh@bgsu.edu, 419-372-3119).

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