Welcome to …

(CO)^{2}MP Elementary

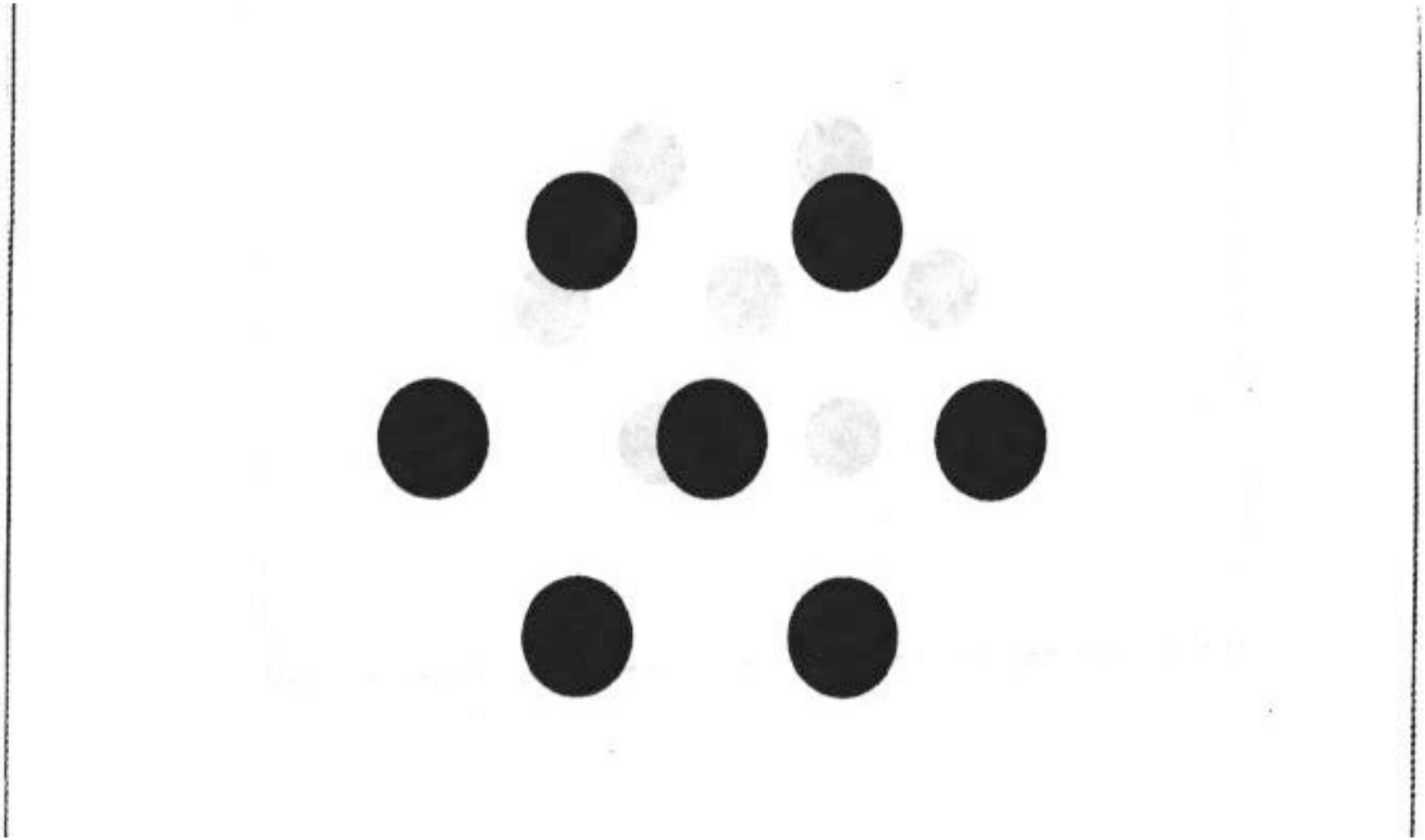
Common Core for Mathematical Proficiency in Elementary Schools

Agenda

- Number Talk
- Skip Counting?
- Multiplication- Expanding our Meanings
- Lunch
- Connecting Arithmetic to Algebra
- Weighing Objects and Balancing Blocks
- Developing K-5 Students Idea of Equivalence
- Reflection

Morning Warm-Up

Determine the number of dots without counting each dot.



Algebra –Multiplication

Discuss: Is skip counting multiplication? Give reasons for why or why not with examples.

Algebra –Multiplication

Thinking about student thinking through problem solving:

The 3rd and 5th grades tied to win the school fundraiser competition. As a reward, the PTA decided to give the four 3rd grade classrooms $600 and the six 5th grade classrooms $800 to spend on classroom supplies. Do 3rd grade classes get more, less, or the same amount of money as 5th grade classes?

Algebra –Multiplication

Multiplicative reasoning – is coordinating and reasoning about composite units (i.e., 1 pan of brownies as three units of a third; a unit of 3 as three units of 1 and 9 as three of the 3 units); involves relative comparisons (Inhelder & Piaget, 1964)

In contrast with…

Additive reasoning – is coordinating and reasoning about a single unit; involves absolute comparisons (a one-to-one comparison)

Algebra –Multiplication

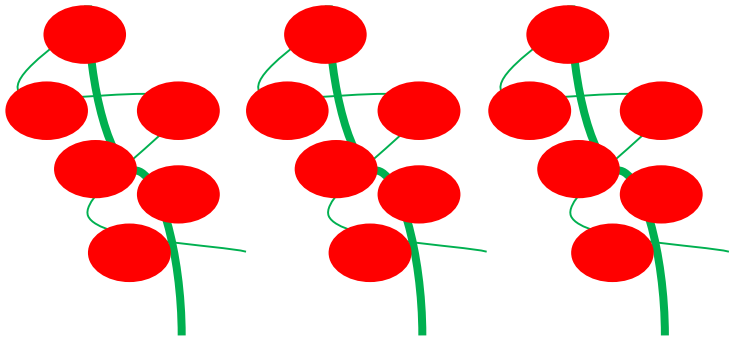
Let's explore the types of multiplication problems:

- Multiplicative Comparison
- Equal-groups
- Arrays and Areas
- Counting

Algebra –Multiplication

Equal-groups:

Jean has 3 tomato plants. There are 6 tomatoes in each plant. How many tomatoes are there all together?

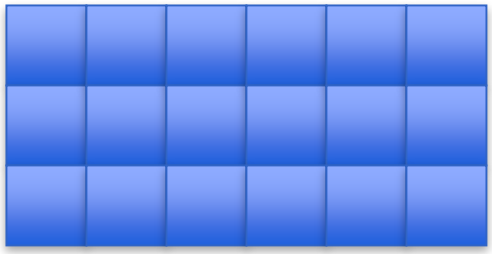


3 × 6 = 18 tomatoes

Algebra –Multiplication

Array and Area:

A baker has a fudge pan that measures 3 inches on one side and 6 inches on the other side. If the fudge is cut into square pieces with 1 inch on the side, how many pieces of fudge does the pan hold?



3 × 6 = 18 pieces of fudge

Algebra –Multiplication

Multiplicative Comparison:

"*n* times as many" or "*n* times as much"

The kangaroo in the zoo is 6 feet tall. The giraffe is 3 times as tall as the kangaroo. How tall is the giraffe?

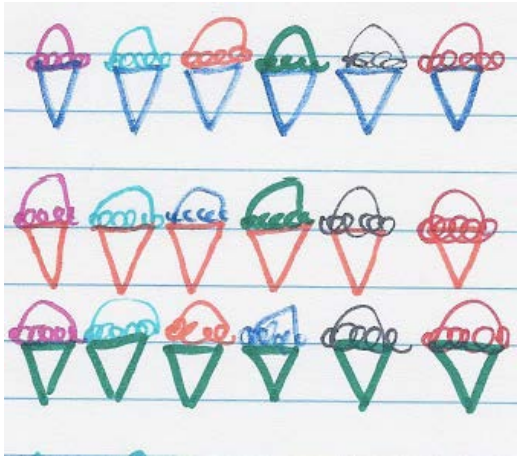
Giraffe's height

Kangaroo's height

Algebra –Multiplication

Counting:

The ice cream store has 3 types of cones and 6 flavors of ice cream. How many different desserts of one cone with one scoop of ice cream can they make?



C_1F_1	C_1F_2	C_1F_3	C_1F_4	C_1F_5	C_1F_6
C_2F_1	C_2F_2	C_2F_3	C_2F_4	C_2F_5	C_2F_6
C_3F_1	C_3F_2	C_3F_3	C_3F_4	C_3F_5	C_3F_6

$3 \times 6 = 18$
kinds of one scoop cones

Algebraic Notation for Multiplication Problem Solving

Using the meaning of multiplication for whole numbers and knowledge of algebra organize each problem under the algebraic equation that best fits the problem. Justify your answer.

A. $a \times b = ?$

B. $? \times b = p \ \& \ p \div b = ?$

C. $a \times ? = p \ \& \ p \div a = ?$

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 + 3 = ?$	$? \times 6 = 18$, and $18 + 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Algebra –Multiplication

Teachers should provide experiences with multiple interpretations of multiplication and division, and solution strategies (with whole numbers and rational numbers)

Teachers can build on students' understanding and solution strategies

Then, by allowing students to share their solution strategies students can build deeper understanding for the abstract representations of algebra

Lunch

Connecting Arithmetic to Algebra

"While first learning how to integrate generalizing into instruction, many teachers use a classroom routine that provides a regular structure for this kind of investigation. They allocate a regular time several days each week – 10 to 15 minutes or longer – to noticing generalizations. Creating this regular routine helps students develop the habit of noticing, articulating, and investigating generalizations."

- Russell, Shifter, Bastable (2011)

Connecting Arithmetic to Algebra

Read page 16 to page 22.

As you are reading, highlight, underline, or make note of things you find significant or meaningful.

When your group is finished, take some time to discuss these things you highlighted, underlined, or made a note of.

Connecting Arithmetic to Algebra

Focus Question #2 (pg. 23)

This section of Chapter 2 details some examples of routines that teachers can use to provide opportunities for their students to investigate generalizations.

•If you have used such routines previously, what is the same and what is different about the examples in the chapter in comparison to your past experiences?

•What routine might you try and what is it you would like to learn by implementing it?

Weighing Objects

- Individually work through the “Weighing Objects” packet of problems.
- Share your solutions and explanations with a partner.

Weighing Objects

- Use these questions to discuss in your small group:
- Did you find anything particularly difficult in solving the problems?
- What do you notice about the progression of the problems?
- Whole group solution sharing and discussion

Balancing Blocks

- Complete the “Balancing Blocks” packet individually.
- Share your solution strategies and explanations with a partner.
- Whole group sharing and discussion
- What was most difficult about solving these problems?
- How were these problems different from “Weighing Objects” problems?

Equations as Statements of Equivalence

- The equals sign is a symbol that represents a relationship of equivalence.
- Equations can be reasoned about in their entirety rather than as a series of computations to execute.
- Equations can be used to represent problem situations.

Different Meanings of Variable

- Used to represent a fixed but unknown number
- Used to represent a quantity that varies in relation to another quantity
- Used to represent a parameter
- Used to represent an arbitrary or abstract placeholder in an algebraic process

Applet

- Demonstrate use of the “Navigating Algebraic Thinking” applet
- Write number sentences for the examples.

Time of Reflection

Take a few moments to reflect on our time of thinking and learning today.

-- Jot down the meaningful and significant things you thought about.

-- Jot down the ways you thought mathematically and pedagogically.

-- Jot down how you contributed to our shared community of professionals.

Stay Safe

- Please help us put the room in proper order.
- Please leave your name tags for next time.