



Making It Happen

**A Guide to Interpreting and Implementing
Common Core State Standards
for Mathematics**



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

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Foreword

Thirty years after NCTM's publication of the groundbreaking report *An Agenda for Action* (1980), and twenty years after NCTM's release of *Principles and Standards for School Mathematics* (2000), the Common Core State Standards have been published by the National Governors Association Center for Best Practices and the Council of Chief State School Officers. The Common Core State Standards have been developed to provide more clarity on what students are expected to learn, in an effort to make mathematics education more consistent across states in our nation and to guide teachers and parents in preparing students for the challenges of the workplace or postsecondary study.

The Common Core State Standards appeared in 2010, after three decades of prior NCTM Standards work. *An Agenda for Action* proved to be a remarkably powerful launch for the influential NCTM Standards movement, which described what school mathematics programs should look like. A succession of NCTM publications have since reinforced and refined that vision. In 1989, *Curriculum and Evaluation Standards for School Mathematics* expanded the recommendations in *An Agenda for Action* and provided a major impetus for states and school districts to develop curriculum guidelines. Ten years later, *Principles and Standards for School Mathematics* added Process Standards to Content Standards for school mathematics and clarified and elaborated the 1989 Standards. In 2006, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* extended that work and described the most significant concepts and skills at each grade level. And in 2009, *Focus in High School Mathematics: Reasoning and Sense Making* detailed NCTM's vision of the critical roles of reasoning, communication, connections, and mathematical problem solving in the teaching and learning of all content in secondary mathematics.

As a means of supporting teachers, districts, and states in implementing the Common Core State Standards, NCTM now presents *Making It Happen: A Guide to Interpreting and Implementing Common Core State Standards for Mathematics*. This publication connects the Common Core Standards with the Council's thirty-year track record of success in the development of standards for school mathematics. Many of the aims and goals of the Common Core State Standards Initiative are consistent with those of NCTM's preceding work—to provide all students with mathematics education of the highest quality. That will always be at the heart of NCTM's mission. *Making It Happen* is another step in realizing that vision.

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Preface

The Common Core State Standards Initiative (CCSSI) is a state-led effort coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) to develop a common core of state standards for proficiency in English language arts and mathematics for kindergarten through grade 12. The National Council of Teachers of Mathematics (NCTM) expressed its support in June 2010 for the goal and intent of the Common Core State Standards for Mathematics effort, which aimed to produce a common set of standards for learning across the nation, strengthening the continuity of instruction and the coherence of auxiliary systems (e.g., assessment systems, textbooks) as well as facilitating the growing number of transfers of students across school boundaries.

The Common Core State Standards (CCSS) 2010) are intended to address the highly challenging task of defining the knowledge and skills that students should gain as they progress from kindergarten through grade 12 to ensure that they will graduate from high school able to succeed in introductory-level, credit-bearing academic college courses and in jobs in the workforce. States are asked to adopt the Common Core Standards in their entirety and include the core in at least 85 percent of the state's standards in English language arts and mathematics.

The *Common Core State Standards for Mathematics (CCSSM)* build on many years of work by NCTM to define the mathematics that students need to know and be able to do. NCTM became the first national organization to develop content standards in mathematics with its publication of *Curriculum and Evaluation Standards for School Mathematics* (1989) and the subsequent revision and update, *Principles and Standards for School Mathematics* (2000). More recently, the Council published *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006) to address issues of curricular consistency, coherence, and depth in the school years before high school. In 2009, NCTM published *Focus in High School Mathematics: Reasoning and Sense Making* to articulate a vision for high school mathematics based on reasoning and sense making and encouraging students to develop reasoning habits throughout their mathematics learning.

While CCSSM was in development, NCTM organized review panels to provide extensive comments and detailed suggestions to the developers and writers on every successive draft after the first. Individual NCTM members served on the writing and validation committees. The Council diligently monitored the development of CCSSM and advised NGA and CCSSO throughout the process. NCTM pointed out common ground between the common core standards and the Council's own K–12 Standards publications while also articulating areas where the two sets of standards diverged.

NCTM's underlying philosophy and its foundational publications offer teachers support in their implementation of CCSSM. Teachers can use the Council's continually growing collection of resources to flesh out CCSSM and fully engage students in doing mathematics. The extensive collection of materials that NCTM



has developed to support and demonstrate the Council's Standards can similarly inform implementation of CCSSM. NCTM can also help fill in some gaps in CCSSM. NCTM's prekindergarten materials provide guidance for parents and early childhood educators in preparing young children to get the most out of their kindergarten mathematics experiences. NCTM has also developed materials that can be used in conjunction with CCSSM to illustrate effective uses of technology or extend students' experiences into areas such as statistics.

Indeed, NCTM's resources can help ensure that the full vision of CCSSM is achieved. Significant next steps in a reaching the ideal of a common core standards environment will be the development of assessments that are consistent with CCSSM and reflect what NCTM's work has helped to make known about the importance of reasoning and sense making in mathematics.



Introduction

The Common Core Standards for Mathematics (CCSSM), as part of the Common Core Standards Initiative, provide a detailed description of content expectations and mathematical practices for K–12 mathematics programs in the United States. CCSSM is part of a decades-long journey in developing a national vision for school mathematics that prepares all students for future success. The National Council of Teachers of Mathematics (NCTM) has played a leading role in this journey, as evidenced by its pivotal documents and other efforts to articulate a vision, influence policy, and improve practice.

As teachers, teacher leaders, and district and state supervisors face the challenge of interpreting and implementing CCSSM, NCTM stands ready with a variety of resources that can help in meeting the curricular, assessment, instructional, and professional development demands of enacting the shared national vision. This resource guide is a tool for clarifying policy and shaping instruction. Its goal is to help educators adopt and enact a shared position on standards, pedagogical processes, and content foci. It aims to identify and highlight the ways in which NCTM resources can support teachers as they implement and supplement CCSSM in their states. The guide and accompanying database are tools to help educators as they continue to make high-quality mathematics happen in their classrooms and schools.

Organization of the Guide

This guide has five main sections that are designed to help teachers, teacher leaders, and district and state supervisors make sense of CCSSM and implement the vision that NCTM and CCSSM share:

1. A Shared Vision
2. Interpreting the Common Core Standards
3. Emphasizing Mathematical Practices and Processes
4. Implementing the Common Core Standards
5. Using NCTM Resources to Implement CCSSM

The first section, “A Shared Vision,” articulates the key characteristics of school mathematics that are common to CCSSM and NCTM’s efforts. The next two sections delve into CCSSM. The second section, “Interpreting the Common Core Standards,” focuses on developing an understanding of the main content points of CCSSM and how they dovetail with NCTM’s previous work. The third section, “Mathematical Practices and Processes,” probes deeply into the overlap between CCSSM’s Standards for Mathematical Practice and NCTM’s Process Standards. This section also supports the fourth section, “Implementing the Common Core Standards,” which provides a road map to existing NCTM resources that can support teachers as they strive to enact CCSSM-consistent curricula in their classrooms. The fifth and final section, “Using NCTM Resources to



Implement CCSSM,” illustrates how educators can use combinations of NCTM resources to address challenges that arise as they work to realize the shared vision. The examples address curriculum, instruction, assessment, and professional development.

Building on NCTM Publications

In framing its recommendations, this guide builds on three major individual publications and one major series from NCTM:

- *Principles and Standards for School Mathematics* (NCTM 2000). This landmark publication is the most recent iteration of NCTM’s Standards for school mathematics and provides standards for four grade bands (pre-K–grade 2, grades 3–5, grades 6–8, and grades 9–12), along with principles for school mathematics programs.
- *Curriculum Focal Points for Prekindergarten through Grade 8: A Quest for Coherence* (NCTM 2007). This publication presents grade-level content recommendations that build on *Principles and Standards*, including focal points and connecting ideas for each grade.
- *Focus in High School Mathematics: Reasoning and Sense Making* (NCTM 2009). This publication offers guidance for focusing the high school mathematics program on reasoning and sense making. It does not provide specific grade-level content recommendations.
- *Essential Understanding Series*. With the first volumes appearing in 2010, this series of sixteen books aims to articulate what teachers of mathematics in prekindergarten through grade 12 need to know to support their work in areas of mathematics that are difficult to teach, challenging to learn, and critical to students’ success in school and beyond.

NCTM has developed extensive resources in support of these efforts, including the e-Standards, Navigations Series, and Illuminations website to complement *Principles and Standards*, the supplemental grade-band and grade-level books that flesh out *Curriculum Focal Points*, and the topic books and related resources developed to elaborate ideas in *Focus in High School Mathematics*. These materials and many other books, journal articles, and print and electronic resources produced by NCTM are incorporated into the suggested resources.



A Work in Progress

NCTM has produced or has in development a wealth of resources on mathematics, classroom practice, and research that have the potential to be useful in interpreting or implementing CCSSM. Many of these resources do not currently appear or are only partially sketched in the charts in *Making It Happen*. Some resources— notably, forthcoming volumes in NCTM’s Essential Understanding Series—are identified as “projected.”

Users should note that NCTM presents *Making It Happen* as a road map that is still under construction. Many routes have yet to be drawn in, and many paths have yet to be linked up. However, recognizing the need for such a tool in the field of mathematics education, the Council did not want to delay in making it available to teachers, administrators, and others who are working with CCSSM now.

NCTM intends to fill in, flesh out, and expand this road map over time. Users will see it change and grow, and they will have access to later, updated versions of *Making It Happen*.



A Shared Vision

CCSSM builds on the vision and recommendations in NCTM's publications.

Focusing only on the skills in the specific standards defined for a grade level will not lead students to develop the mathematical competence that they need for future success. Important mathematics includes both practices or processes and content, organized in a coherent and focused way.

The Common Core State Standards for Mathematics (CCSSM) provide direction for K–12 mathematics education and to a great extent build on NCTM's past efforts. However, CCSSM serves a different purpose from NCTM's publications. The Council's Standards and related work have focused on *articulating a vision* for school mathematics, providing guidance for the development of state and local standards. CCSSM builds on that vision by *articulating state standards* in a detailed form that can be immediately adopted and implemented as state-level curriculum frameworks. Despite some differences in the specific content emphases and recommendations, CCSSM and NCTM's standards-related work share a common vision.

The Curriculum Principle in *Principles and Standards for School Mathematics* (NCTM 2000) succinctly captures this perspective: “A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well articulated across the grades” (p. 14). One common characteristic of the shared vision of CCSSM and NCTM is the need to have a curriculum that is *focused*. CCSSM articulates this need through the identification of *critical areas* for kindergarten through grade 8 (pp. 9, 13, 17, 21, 27, 33, 39, 46, 52, respectively). These areas overlap with the Curriculum Focal Points (pp. 11–20) that NCTM identifies in *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM 2006). The focus in both cases is on “big ideas” of mathematical content, with specific content clusters targeted at each grade level. NCTM's *Focus in High School Mathematics: Reasoning and Sense Making* (NCTM 2009) suggests *key elements* (pp. 21, 31, 41, 55–56, 73–74, respectively) within five content strands in grades 9–12 that serve as a focus for reasoning and sense making.

CCSSM and NCTM's work both emphasize the importance of a coherent curriculum. CCSSM, *Curriculum Focal Points*, and *Focus in High School Mathematics*, like *Principles and Standards for School Mathematics*, emphasize the need for a mathematics curriculum that is more than a collection of topics distributed across the years. Ideas at one grade level must build on those from previous years and form the basis for ideas in later years. Lessons must be designed to engage students with particular mathematical ideas in the service of big ideas. The organization of the content across grades must reflect both what is mathematically meaningful and what is known from research and practice about learning mathematics.

CCSSM and NCTM share a view of important mathematics that includes both content and mathematical practices or processes. The third section of this publication, “Emphasizing Mathematical Practices and Processes,” provides an extended discussion of CCSSM's highlighting of mathematical practices and NCTM's focus on mathematical processes. Although slightly different in their details, these practices and processes are both essential mathematics to be learned and ways in which students engage in learning other mathematics content.

CCSSM sets grade-specific standards but does not address issues related to how those standards might be implemented—nor was doing so a part of the charge for CCSSM. NCTM has long argued that mathematics



NCTM's experience in promoting the broader vision of school mathematics will be an important ingredient in successful implementation of CCSSM.

curriculum must be considered as one part of a bigger picture of a strong mathematics program that builds on well-aligned and coordinated attention to curriculum, assessment, and instruction. Professional development is a means for continued refinement of school programs and for enhancement of educators' capacity to bring those programs to life for students. NCTM has long been concerned about and involved in professional development, and thus its work offers a means to support teachers in their implementation of CCSSM.

Professional development that effectively supports teachers in their implementation of the shared vision of CCSSM and NCTM includes opportunities for teachers to develop their personal mathematical understanding. To interpret and implement a coherent and focused curricular vision that organizes mathematics content around critical areas and engages students in mathematical practices and processes requires teachers to have an essential understanding of mathematics that encompasses and transcends what they expect their students to know and be able to do. In thinking about critical areas and processes for students, teachers benefit from an understanding of big ideas that extend beyond them. NCTM's Essential Understanding Series articulates the big ideas of both mathematical content and mathematical processes that are essential for teachers to know to be able to interpret and implement the shared vision.

NCTM's experience in promoting the broader vision of school mathematics will be an important ingredient in successful implementation of CCSSM. *Principles and Standards, Curriculum Focal Points, Focus in High School Mathematics*, and the volumes in the Essential Understanding Series—along with many other NCTM publications and services—constitute a rich set of tools to help teachers and others interpret and implement the shared vision.



Interpreting the Common Core Standards

Understanding the Common Core State Standards for Mathematics (CCSSM) includes knowing what CCSSM contains and how it relates to familiar guidelines and resources. NCTM materials are useful in making sense of CCSSM's identification of *critical areas*, *domains*, and *clusters of standards* and connecting these ideas to NCTM's prior work. To demonstrate relationships between NCTM's resources and CCSSM, this section juxtaposes CCSSM's expectations for kindergarten through high school with corresponding elements in NCTM's *Principles and Standards for School Mathematics*, *Curriculum Focal Points for Prekindergarten through Grade 6 Mathematics*, and *Focus in High School Mathematics: Reasoning and Sense Making*. Components of volumes in NCTM's Essential Understanding Series help to place CCSSM's content for students in the broader context of mathematics content for teachers.

Prekindergarten–Grade 8

CCSSM identifies *critical areas* of mathematics that students are expected to learn each year from kindergarten through grade 8. These are the big ideas of school mathematics at each grade level. (CCSSM does not address prekindergarten mathematics.) More learning time should be devoted to the critical areas than to other content. In other parts of this guide, the standards, which CCSSM organizes into clusters that fall under domains, are tied to critical areas (pp. 9, 13, 17, 21, 27, 33, 39, 46, 52). The critical areas that CCSSM identifies for kindergarten through grade 8 align closely with the Focal Points identified for these levels in *Curriculum Focal Points*. Both critical areas and Curriculum Focal Points are sets of mathematical ideas that students are expected to learn at an indicated grade level. In CCSSM, clusters typically correspond to NCTM Content Standards and one or more Expectations associated with particular Standards, as identified in *Principles and Standards*. Portions of books in the Essential Understanding Series articulate mathematical understanding that can be useful to teachers who are working to help students meet standards of CCSSM.

For each grade level, this guide provides a chart that details these relationships among CCSSM's critical areas, the Content Standards and Expectations in *Principles and Standards*, the Focal Points in *Curriculum Focal Points*, and the mathematics for teachers treated in the Essential Understanding Series. Each chart is organized according to the template on the next page.

In CCSSM, critical areas lie alongside domains, which contain clusters of specific standards.



Interpretation Chart for Each Level, Pre-K–Grade 8

CCSSM Critical Area	<i>Principles and Standards</i>	<i>Curriculum Focal Points</i>	<i>Essential Understanding Series</i>
	For information about instructional goals related to the mathematics content in this critical area	For information about how this content appears within an example of a focused curriculum proposed by NCTM	For an articulation of mathematical understanding that is essential for teachers working in this critical area

Access to the complete set of appendices is available at www.nctm.org/nih.

An educator who is trying to interpret CCSSM might be most likely to ask, “If my current curriculum was written with NCTM Standards and Focal Points in mind, how do the Common Core Standards relate to those?” This guide answers that question logically by looking first at a CCSSM critical area and then seeing how it relates to *Principles and Standards* (the broadest perspective) and *Curriculum Focal Points* (a more targeted perspective). Therefore, the columns in the chart appear in that order from left to right. Because books in the Essential Understanding Series address the placement and development of the content related to the CCSSM critical area, the chart places them in the last column, as a transition to professional development and implementation issues. To view the chart for a particular grade level, see Appendix A. Online users can click on the desired grade level to follow a link to the chart:

Prekindergarten	Grade 3	Grade 6
Kindergarten	Grade 4	Grade 7
Grade 1	Grade 5	Grade 8
Grade 2		

Grades 9–12

In contrast to the way in which CCSSM presents expectations for kindergarten–grade 8, it does not provide grade-by-grade or course-by-course standards for high school mathematics, nor does it identify or articulate critical areas. Instead, the standards for grades 9–12 are organized by six conceptual categories: number and quantity, algebra, functions, modeling, geometry, and statistics and probability. For each of these categories, other than modeling, CCSSM arranges specific standards into *domains*, each of which contains *clusters* of specific *standards* (example, p. 60). Standards related to modeling are incorporated in other conceptual categories (example, p. 62).

Conceptual categories contain domains, which contain clusters of specific standards.



Although *Principles and Standards* also arranges its content guidelines by grade band, its organization is quite different from that of CCSSM. *Principles and Standards* presents five overarching Content Standards that are common across the grade bands, and it breaks each Standard into several areas and gives specific Expectations for each area (example, p. 395).

Focus in High School Mathematics provides guidance about ways to integrate reasoning and sense making into instruction and learning in five content strands across grades 9–12. Reasoning and sense making in each of these strands includes several characteristic Key Elements.

The following chart illustrates the very different organizations of CCSSM, *Principles and Standards*, and *Focus in High School Mathematics*. None of the three gives specific grade- or course-level recommendations or critical areas for grades 9–12, although possible “pathways,” along which content can be organized, appear in Appendix A of CCSSM.

Organizational Elements of CCSSM, *Principles and Standards*, and *Focus in High School Mathematics*

CCSSM Conceptual Categories	<i>Principles and Standards</i> Content Standards	<i>Focus in High School Mathematics</i> Content Strands
Number and Quantity --- Algebra Functions Modeling Geometry Statistics and Probability	Number and Operations Measurement Algebra <i>(Included in Algebra)</i> --- Geometry Data Analysis and Probability	Number and Measurement <i>(Included in Number and Measurement)</i> Algebraic Symbols Functions --- Geometry Probability and Statistics
Organized by— - Domains - Clusters - Standards	Organized by— - Areas - Expectations	Organized by— - Key Elements



This guide provides charts, arranged according to the template below, to show the connections in the content of the three sets of guidelines. Five such charts organize information about conceptual categories identified in CCSSM. (The charts omit modeling, the sixth CCSSM conceptual category, which does not have specific standards.).

Interpretation Chart for Each Conceptual Category, Grades 9–12

CCSSM Cluster of Standards Identified by Domain	<i>Principles and Standards</i>	<i>Focus in High School Mathematics</i>	<i>Essential Understanding Series</i>
	For information about instructional goals related to the mathematics content in this cluster	For information about promoting reasoning and sense making in this cluster	For an articulation of mathematical understanding that is essential for teachers working in this cluster

The content in each chart is organized by clusters within the domains for a conceptual category, as indicated in the first column. Rows in the chart are arranged by either a cluster or a domain, depending on its “grain size.” Some domains are relatively specific (e.g., “Vector and Matrix Quantities”), whereas some clusters are quite general (e.g., “Interpret functions that arise in applications in terms of the context”).

The second column provides corresponding Expectations from *Principles and Standards*, organized by areas. The third column identifies Key Elements from *Focus in High School Mathematics* that may be useful in understanding possibilities for incorporating reasoning and sense making in the content related to the row’s cluster.

The fourth and final column indicates a portion of one or more books in the Essential Understanding Series that articulate mathematical understanding that can benefit teachers in developing the content of this conceptual category. The category Number and Quantity has no corresponding Essential Understanding book, but components of several books in the series are closely related, including [chapter 1 of the 9–12 proof and proving book](#) and the vertical articulation issues in [chapter 2 of several 6–8 books](#).

To view the chart for a particular conceptual category, see Appendix A. Online users can click on the conceptual category to follow a link to the chart. (As noted above, no chart is given for the conceptual category of modeling. Standards on modeling are incorporated in other conceptual categories, and “model with mathematics” is a mathematical practice.)

Number and Quantity	Geometry
Algebra	Probability and Statistics
Functions	



Emphasizing Mathematical Practices and Processes

Although CCSSM places significant emphasis on the content that students need to learn in each grade, educators should not regard that content narrowly, as a checklist. Rather, CCSSM stresses that it is essential to connect its Standards for Mathematical Content with its Standards for Mathematical Practice. CCSSM's Standards for Mathematical Practice are closely related to NCTM's Process Standards, as elaborated in *Principles and Standards for School Mathematics* (NCTM 2000). This section explores the relationship between the two.

CCSSM's View of Mathematical Practices

CCSSM emphasizes the need to look beyond specific skills to “processes and proficiencies’ with longstanding importance in mathematics education” (p. 6), explicitly echoing the thinking behind the Process Standards articulated in *Principles and Standards*. The introduction to CCSSM (CCSSI 2010, p. 4) juxtaposes the nature of content standards and the importance of one of the processes—reasoning:

[CCSSM’s] Standards define what students should understand and be able to do in their study of mathematics.... One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from.

The Standards for Mathematical Practices (CCSSI 2010, pp. 6–8) provide more detailed descriptions of the proficiencies with process that CCSSM advocates for students. As enumerated by CCSSM, students need to be able to do the following:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Process Standards in *Principles and Standards*, as well as the Reasoning Habits in Focus in *High School Mathematics*, provide important insights into the Standards for Mathematical Practice in CCSSM.



CCSSM elaborates each of these eight standards in a paragraph and explains that together they describe “varieties of expertise that mathematics educators at all levels should seek to develop in their students” (CCSSI 2010, p. 6). CCSSM also emphasizes the need to connect the Standards for Mathematical Practices with the Standards for Mathematical Content.

NCTM’s Longstanding Emphasis on Process

As previously noted, CCSSM makes explicit reference to the Process Standards enunciated in *Principles and Standards*. In fact, when NCTM released this landmark publication in 2000, the Council had already been focusing on mathematical processes for a long time. *An Agenda for Action* (NCTM 1980) emphasized the centrality of mathematical problem solving, and *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) described Process Standards for three grade bands.

Principles and Standards describes five Content Standards and five Process Standards that NCTM urges educators to make the focus of school mathematics, prekindergarten–grade 12. The Process Standards—for Problem Solving, Reasoning and Proof, Communication, Connections, and Representation—“highlight ways of acquiring and using content knowledge” (NCTM 2000, p. 29), thus underscoring the close interrelationship of process and content also expressed in CCSSM.

Principles and Standards identifies several major areas of emphasis for each Process Standard, and a chapter devoted to each grade band discusses characteristics of these processes at that level. In addition, NCTM’s Navigations Series includes a number of volumes that explore students’ development of skill in using particular processes identified by the Process Standards.

NCTM has released two publications since 2000 to provide additional guidance related to *Principles and Standards: Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM 2007) and *Focus in High School Mathematics: Reasoning and Sense Making* (NCTM 2009). Although the Council’s intention with *Curriculum Focal Points* was primarily to provide a concise and coherent description of important content for each grade level encapsulated in Focal Points and Connections (links from the Focal Points to other, related ideas), attention to process was embedded within all of those grade-level descriptions.

In contrast, the companion volume for high school, *Focus in High School Mathematics*, maintains a clear focus on the Process Standards—particularly, the Problem Solving Standard and the Reasoning and Proof Standard. This publication provides a detailed list of Reasoning Habits, organized into four major categories: analyzing a problem, seeking and using connections, implementing a strategy, and reflecting on a solution. Recommending that students develop these as a part of the high school mathematics program, the book presents a set of chapters that describe possibilities for embedding opportunities for reasoning and sense making in five major content strands, with examples of the development of the Reasoning Habits.



Comparison of the CCSSM and NCTM Approaches to Process

A comparison of the Mathematical Practices set forth in CCSSM with both the Process Standards developed in *Principles and Standards* and the Reasoning Habits described in *Focus in High School Mathematics* reveals general alignment between the goals of CCSSM and NCTM's efforts to promote process in mathematics. Table 1 provides an overview. Note that the comparisons are at a very general level, and only some aspects of a Process Standard or category of Reasoning Habits corresponds to a particular Standard for Mathematical Practice.

Table 1. General comparison of attention to process in CCSSM, *Principles and Standards*, and *Focus in High School Mathematics*.

CCSSM Standards for Mathematical Practices	<i>Principles and Standards</i> Process Standards	<i>Focus in High School Mathematics</i> General Categories of Reasoning Habits
1. Make sense of problems and persevere in solving them.	Problem Solving Communication Representation	Analyzing a problem Seeking and using connections Implementing a strategy Reflecting on a solution
2. Reason abstractly and quantitatively.	Problem Solving Reasoning and Proof	Analyzing a problem
3. Construct viable arguments and critique the reasoning of others.	Reasoning and Proof Communication Representation	Analyzing a problem Implementing a strategy Reflecting on a solution
4. Model with mathematics.	Problem Solving Reasoning and Proof Connections Representation	Analyzing a problem Seeking and using connections Reflecting on a solution
5. Use appropriate tools strategically.	Problem Solving Representation	Analyzing a problem Reflecting on a solution
6. Attend to precision.	Problem Solving Communication	Analyzing a problem Reflecting on a solution
7. Look for and make use of structure.	Problem Solving Reasoning and Proof Connections	Analyzing a problem Implementing a strategy
8. Look for and express regularity in repeated reasoning.	Problem Solving Connections	Analyzing a problem Implementing a strategy



In fact, differences in how the three documents approach mathematical processes are significant. In contrast to the fairly general Process Standards articulated in *Principles and Standards*, CCSSM provides rather specific statements of the processes in which students should engage. *Principles and Standards* paints a broader view of the role of mathematical processes, incorporating some processes that are not explicitly addressed in CCSSM, such as building new mathematical knowledge through problem solving, the first area described within the Problem Solving Standard in *Principles and Standards*.

Although the categories of Reasoning Habits in *Focus in High School Mathematics* are broader than the Mathematical Practices in CCSSM, the individual Reasoning Habits that *Focus in High School Mathematics* details are generally a close match in level of specificity for the descriptions of the Mathematical Practices in CCSSM. For example, in describing Mathematical Practice Standard 3, “Construct viable arguments and critique the reasoning of others,” CCSSM states that students “make conjectures and build a logical progression of statements to explore the truth of their conjectures” (CCSSI 2010, p. 6). Similarly, under the category “Reflecting on a solution,” *Focus in High School Mathematics* suggests the Reasoning Habit “making logical deductions based on current progress, verifying conjectures, and extending initial findings” (NCTM 2009, p. 10).

Despite similarities between the Reasoning Habits in *Focus in High School Mathematics* and the Mathematical Practices in CCSSM, the two lists are organized very differently, and each reveals some unique perspectives. For example, CCSSM places somewhat greater emphasis on precision, as evidenced in Mathematical Practice Standard 6, “Attend to precision,” which includes the suggestion that students should “try to use clear definitions in discussion with others and in their own reasoning” (CCSSI 2010, p. 7). In contrast, *Focus in High School Mathematics* gives more attention to Reasoning Habits related to statistics, such as “deciding whether a statistical approach is appropriate” (NCTM 2009, p. 9), a habit identified in the category “Analyzing a Problem.” Reasoning Habits related to statistics are discussed in more depth in *Focus in High School Mathematics: Statistics and Probability* (Shaughnessy, Chance, and Kranendonk 2009), a volume in an NCTM companion series supporting and extending *Focus in High School Mathematics*.

A more detailed version of table 1 is presented in Appendix B, which shows the connections among the three documents at a finer level. In Appendix B, the statements from the descriptive commentary in the Standards for Mathematical Practice in CCSSM are matched with the areas of emphasis within the Process Standards from *Principles and Standards* and the specific Reasoning Habits from *Focus in High School Mathematics*.

Although CCSSM, *Principles and Standards*, and *Focus in High School Mathematics* provide unique ways of describing mathematical processes, the three are united in their emphasis on the importance of mathematical processes, both as a goal and as a means of learning mathematics. The sections that follow give particular examples to illustrate possibilities for using NCTM resources to address the Standards for Mathematical Practices in support of the Standards for Mathematical Content, thus implementing CCSSM.



Implementing the Common Core Standards

Implementing the Common Core State Standards for Mathematics (CCSSM) involves emphasizing mathematical practices and highlighting big ideas in a focused and coherent mathematics curriculum. This section contains separate discussions for prekindergarten–grade 8 and grades 9–12 because CCSSM treats these two grade spans in very different ways.

Prekindergarten–Grade 8

CCSSM defines a *cluster* as a group of related standards, and the cluster level is a reasonable starting point for consideration of CCSSM for prekindergarten–grade 8. Appendix C contains one chart for each grade level from kindergarten through grade 8, like the sample chart shown below.

Implementation Chart for Each Level, K–Grade 8

CCSSM Standard Cluster in Grade	Related CCSSM Critical Area in Grade	Sample NCTM Resources
<Category>		

Each chart begins with the cluster descriptors from the CCSSM overview (example, p. 5) for that grade level. The second column of the chart identifies, to the extent possible, a critical issue at that grade level that serves as a mathematical content “umbrella,” under which the cluster fits or that contains some mathematical component to which that cluster can contribute. The third column of the chart contains several NCTM resources that would be useful in addressing the cluster. Each entry in this column includes a note about how the resource connects with the CCSSM cluster.

To view the chart for a particular grade level, see Appendix C. Online users can click on the desired grade level to follow a link to the chart. (Note that CCSSM does not include standards for prekindergarten.)

Kindergarten	Grade 3	Grade 6
Grade 1	Grade 4	Grade 7
Grade 2	Grade 5	Grade 8

Grades 9–12

Although CCSSM does not differentiate grades 9–12 by course or grade, it provides a number of model pathways in Appendix A, “Designing High School Mathematics Courses Based on the Common Core State



Standards.” These pathways are intended to assist states, districts, and teachers in implementing CCSSM, and each pathway represents a different way in which the standards can be organized across a high school curriculum. These pathways are only examples, and CCSSM anticipates that others will be developed. However, the given pathways provide a useful starting point for demonstrating how NCTM resources could support teachers in their implementation efforts.

A pathway consists of a sequence of courses, each divided into content units that address one or more clusters of standards. However, not all standards in a cluster are addressed within a single unit—or even a single course. Moreover, some standards may be fully addressed only over a range of units and courses. Because each pathway may be organized quite differently from others with respect to the courses and the units within them, the focus of this analysis is on how the standards might be addressed progressively within a cluster across the various pathways. These progressions, from students’ initial exposure to their intermediate experience to their final mastery of the content or practice described in the standards within a cluster, provide ways of showing how the mathematical content within a given cluster might grow across the curriculum and how NCTM resources might be used to support that growth.

The implementation charts for high school in Appendix C are organized according to the conceptual categories defined in CCSSM for grades 9–12, with each chart addressing one conceptual category. The first column contains the domains and clusters addressed within a conceptual category, as shown below.

Implementation Chart for Each Conceptual Category, Grades 9–12

Domain/Cluster	Progressions of Standards in Pathways			NCTM Resources
	Initial	Intermediate	Final	
<domain>				
<cluster>	<list of standards and notes>	<list of standards and notes> (if needed)	<list of standards and notes> (if needed)	<list of NCTM electronic and print materials and services that address one or more aspects of the cluster>

Columns 2–4 show possible progressions of the standards within the clusters across the pathways. Because of the different ways in which the standards can be addressed in units or courses, the standards in the “Initial” column do not necessarily fall within the first course in a pathway. Likewise, the standards in the “Intermediate” and “Final” columns may occur within the same course as the standards in the “Initial” column or in a subsequent course. A horizontal bar within a cell indicates a further subdivision of the standards within that cell that occurs in at least one of the pathways.



The last column in each chart presents selected NCTM resources that might be useful in thinking about a particular cluster. These resources include discussions in *Principles and Standards and Focus in High School Mathematics*, interactive activities from Illuminations and the e-Examples for *Principles and Standards*, and research and discussion from other NCTM publications. Where possible, these resources are described in terms of progressions of standards.

To view the chart for a particular conceptual category, see Appendix C. Online users click on the conceptual category to follow a link to that chart. (No chart is given for Modeling. Standards on modeling are incorporated in other conceptual categories and “Model with mathematics” is a mathematical practice.)

Number and Quantity
Algebra
Functions
Geometry
Probability and Statistics