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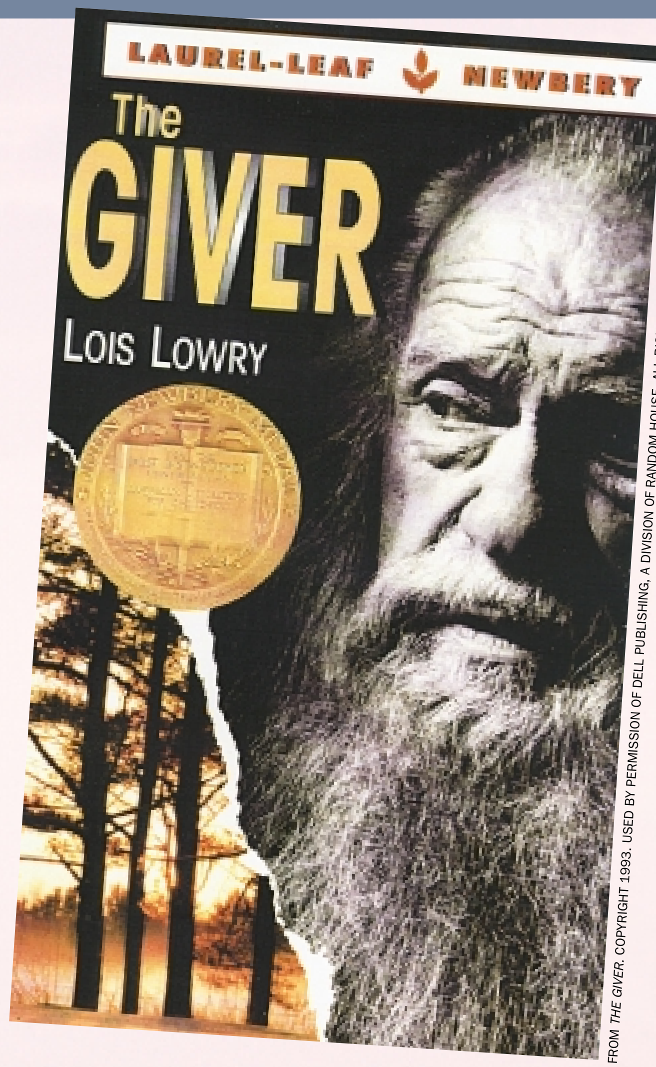
From *The Giver* to *The Twenty-One Balloons*: Explorations with Probability



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BASING AN ACTIVITY IN THE MATHEMATics classroom on an event or a story from children's literature offers a vivid, engaging context for mathematical investigation. The following problem (see **fig. 1**) stems directly from *The Giver,* by Lois Lowry (1993):

> Each class of 50 children in the Community has 25 girls and 25 boys. Assuming that the probability of a woman giving birth to a boy or a girl is the same, how often do you think exactly 25 out of 50 babies will be girls?

Students were asked to predict the result, design a model, and carry out fifty trials to determine the experimental probability. Finally, they were to explain any differences between their predictions and the experimental results.

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When the teacher presented this problem, pairs of seventh graders immediately began to chatter eagerly, making their predictions. They made comments like "Well, we know it will be way less than half the time" and "I wish I had a Pascal's triangle that showed fifty rows." Because these students had had a number of related experiences involving probability, they were confident in their educated guesses. Similarly, the students had little trouble determining an appropriate method for modeling the problem. A few pairs used dice; a few, coins or two-color chips. But most used their calculators to generate random numbers. The class compiled its results to establish the experimental probability, which was about 12 percent, and the students worked together to support their findings by working out the theoretical probability

$$\frac{{}_{50}C_{25}}{2^{50}},$$

since ${}_{50}C_{25}$ represents the number of ways that twenty-five girls can be chosen from a group of fifty

Will Girls and Boys Be Equal?
For use with *The Giver,* by Lois Lowry (1993)

Each class of 50 children in the Community has 25 girls and 25 boys. Assuming that the probability of a woman giving birth to a boy or a girl is the same, how often do you think exactly 25 out of 50 babies will be girls?

I. Model this problem. Do 50 trials. Record your results below.

II. Combine your data with those of your classmates to simulate a large number of years.

III. Now how would you answer the original question, "How often do you think exactly 25 out of 50 babies will be girls?" Explain your answer, using the data your class gathered.

Extension: Suppose the Committee of Elders decided that the Community should expand so that each family has five children. If the babies are randomly assigned to each family, how often will a family have no boys, one boy, etc.?

Related Writing Activity: In the Community, there was Sameness. For example, there was Climate Control—no snow, no sunshine, always the same weather conditions. Write a paragraph describing the one characteristic of Sameness, and create a related problem and solve it.

Fig. 1 Probability problem based on *The Giver*

children and that must be compared with 2^{50}, which is the total number of possibilities for a group of fifty children. To compute the theoretical probability, students needed to rewrite ${}_{50}C_{25}$ as

$$\frac{50!}{(50-25)!(25)!}.$$

Finally, the teacher asked each student to design a problem with a solution similar to this one. Most of their results reflected a clear understanding of probability.

If the teacher had made this assignment without previously having given the students related experiences, the positive results would have been unlikely. Indeed, students need many experiences with probability if they are to develop the skills and confidence that they need for independent explorations involving chance.

Therefore, we take an important look back. The activities that students carried out before the one with *The Giver* were designed to focus on the following concepts:

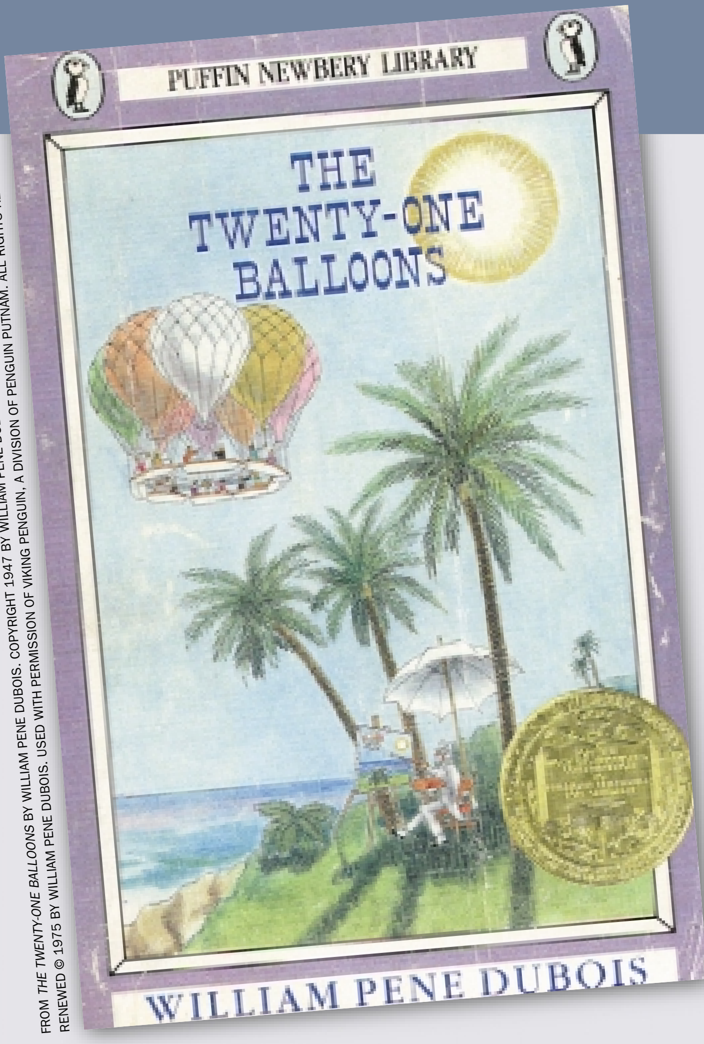
- An informal definition of probability
- The difference between experimental and theoretical probability
- The importance of order in listing possible outcomes
- The importance of determining an accurate outcome set

The students completed each of the carefully ordered activities described subsequently, and, as often happens in middle school classes, some unexpected extensions resulted.

The Initial Probability Experience

AS AN INITIAL WARM-UP, THE TEACHER LED THE class in a brief exploration of basic probability. For most of the students, the discussion was review, but for a few, it was their first exposure to the topic. Students were prompted with the following probes:

- Tell me what you remember about probability.
- What is the range of values for probabilities?
- What is an outcome set?
- In what forms are probabilities expressed? ☞



To make a connection with children's literature, the teacher asked, "Who has read the book *The Twenty-One Balloons,* by William Pene Dubois?" and "Who can summarize the story for us?" In the book, a man discovers a remote, uninhabited island that has numerous diamonds. He selects twenty-five families to join his own and live there permanently; moreover, when he chooses families to go to Krakatoa, he picks only those with exactly one girl and one boy (Dubois 1986).

Once the setting was established, the teacher posed this problem:

> In a family with two children, how often do you think that the family has exactly one girl and one boy?

Next, the teacher distributed the activity sheet (see **fig. 2**) and directed the students to "use a fraction to predict the probability of one girl and one boy in a family of two children."

Students shared their estimates and explained their reasoning. The most common estimate was 1/3. As Alison explained, "There were three ways the family could happen: two boys, two girls, or one boy and one girl. So the probability was one good thing out of three things, or one-third." Other estimates varied widely and were explained with "I thought of families I knew" and "I just took a wild guess." Among the small number who estimated one-half, many said the equivalent of "It felt right." Very few students, one or two in each class, explained their estimates of one-half as Fuller did: "There are four ways to have a family with two children: first, a boy, then another boy; second, a girl, then another girl; third, a girl, then a boy; and fourth, a boy, then a girl. So the probability of one girl and one boy is two-fourths, or one-half." At this point, a lively debate broke out in every class. The teacher did not comment either way. Clearly, most students did not demonstrate intuitive understanding of this situation.

Next, the teacher led a brief discussion about experimental probability and asked each pair of students to develop a model to simulate this problem. Most had no difficulty determining an appropriate model. In each class, all the following models were suggested:

- Flip a coin or a two-color chip.
- Roll a number cube.
- Use a calculator to generate random numbers.

The most unusual model was designed by Garrett and his partner, who counted to three then showed one or two fingers. When the numbers matched, they designated a girl; with no match, they designated a boy. Obviously, this pair had intuition about the underlying mathematics of the problem.

Students described their model in writing, and a volunteer read a description of each suggested method to the class. For example, Yaicha shared, "We will use a two-color chip. We will flip it two times for each family. If it lands on yellow, it will stand for a girl. If it lands on red, it will be a boy. We will do this twenty times." In each class, someone requested a demonstration of how to generate random numbers with a calculator. Seeing a reason to learn this skill, students quickly mastered both the needed keystrokes and the application of random numbers in the context of this problem. Allowing students to choose their own model accommodated a wide range of interest and skill levels easily and subtly.

As each pair worked to complete, record, and total the results for twenty trials, the teacher circulated among them. A few pairs needed help because they thought that it was appropriate simply to generate data to represent forty births: they did not grasp that two events were needed to complete each trial. As Charles later wrote, "We did not understand at first that we were sort of acting out the problem. We found out we needed to roll the cube twice for each family, since there were two kids. Then we started over for the next family." The opportunity to redirect student thinking *on the spot* was an important ingredient in the success of this activity.

After each pair of students totaled its results, the class built on the wall a histogram of the results, compiled the results for the entire class, expressed the results as a fraction, and recorded them on the activity sheet. After the students discussed the combined results, they agreed that the fractional value—and thus the probability of exactly one girl and one boy in a two-child family—was close to 1/2. The teacher then asked students to compare in writing their original predictions with the experimental results. Some students then understood why their prediction was incorrect. As Katherine wrote, "My prediction was totally wrong. This is why. I did not think about a boy and a girl being different from a girl and a boy." Because others were stymied and needed class discussion to clarify their understanding, the teacher asked volunteers to read their explanations and led a brief discussion of theoretical probability.

The class next listed the outcome set for this problem: GG, BB, BG, and GB. As Mignon pointed out in her class, "We have one boy and one girl in my family, and I am the oldest. That is certainly different than if my brother was the older child!" Such examples from people they knew quickly convinced reluc-

Will Girls and Boys Be Equal?

For use with *Twenty-One Balloons* by William Pene Dubois (1986)

When Mr. M chose families to go to Krakatoa, he picked only those with exactly one girl and one boy. How often do you think this happens in a family with two children?

Use a fraction to predict the probability of one girl and one boy in a family of two children. ___________

Design a model you can use to find the experimental probability of one girl and one boy in a family of two children. Describe it below.

Do twenty trials, using your model. Record your results below.

Compare your results with those of your classmates. Combine the data from your entire class, and record the experimental probability (results) below.

How did the results of the experiment compare with your prediction? Why do you think this happened?

Show all the possibilities for a family with two children. Then give the theoretical probability of one girl and one boy in a family of two children.

Write two paragraphs below about this activity.

What we found . . . .

What I learned . . . .

Extension: List all possibilities for a family of four children to determine the theoretical probability of exactly two girls and two boys. Then answer this question: As the number of children in a family increases, what happens to the probability of having the same number of girls and boys? Explain why this happens.

Fig. 2 Probability problem based on *The Twenty-One Balloons*

tant students that BG and GB were two different outcomes. Students then listed the outcome set and found the theoretical probability.

After a discussion about such topics as the reason that the experimental results did not exactly match the theoretical ones and the effects of sample size, each student wrote two summary paragraphs about the activity, titled “What We Found” and “What I Learned.” Sample student responses to these prompts are in **figures 3** and **4.** As the students’ writings suggest, almost all the students could verbalize the results of the activity and believed that they had gained a better understanding of some aspects of probability.

More Probability Activities

THE NEXT ACTIVITIES WERE SLIGHTLY DIFFERent in each class, depending on the written responses and the questions raised by the students during the first activity. The students in every class were asked to find the sample space for a family of four children to determine the theoretical probability of exactly two boys and two girls, or 3/8. They were then asked the following question:

> As the number of children in a family increases, what happens to the probability of having the same number of girls and boys, and why does this happen?

Students had little difficulty transferring their learning from the first problem and using mathematical reasoning to explain the different results. David gave a typical explanation: “The reason the two-children families were different from the four-children families is because when you add more children, you add more possibilities for each family, therefore decreasing your chance to get the same number of boys and girls. For the two-children family, the expected probability is 4/8. For the four-children family, it is 3/8.”

Students next predicted the probability of three boys and three girls in a family with six children. This task initially gave some students trouble, as evidenced by comments like those of Sarah, who wrote, “When you have two children, you have one girl and one boy only half the time. When you have four children, you have six out of sixteen chances of having half girls, half boys. I think that if you have six children, you will have half girls, half boys only one-fourth of the time.” Since listing the outcome set became unwieldy at this point, the teacher introduced Pascal’s triangle. If the rows are numbered starting with row 0 for the apex, the numbers in each row represent the frequency of occurrence of a given outcome. Students used the triangle to find the probabilities for equal numbers of boys and

Write two paragraphs below about this activity.

What We Found . . . . We found that a little bit more than half the time from our experiment that a family with two children would have 1 boy and 1 girl.

I found out that a family would have a half of a chance to get a boy and a girl. I found out the information by getting a die and making the even numbers girls and the odd boys. My results were $\frac{12}{20}$ families would be one boy and one girl. As a class the results equalled $\frac{85}{180}$.

This was a fun activity in that you can get on estimate of what people are trying to find out in the real world. The class found that $\frac{62}{133}$ are Boys and girls in the world.

What We Found . . . . out was that there are more ways than 1 to solve this problem. Also, people could say a boy and a Girl then a Girl and a boy can be 4ths or altogether a half which then makes it a 50%, 25%, and 25% of chances.

Fig. 3 Student responses to the “What we found” prompt

What I Learned . . . . I learned that there are two parts of probability; experimental and theoretical.

I learned that if you look at a simple problem there might be a little more to it than you think. I also learned how to use the calculator in a different way. I got to freshen up in my probability and fractions.

What I Learned . . . . about probability is that you can put probability in 3 ways fraction, percentage, or decimal. I also learned that a G+B + a B+G can be counted as a 1/4 each or a 1/2 together.

You need a large sample to get a good idea what might really happen.

Fig. 4 Student responses to the “What I learned” prompt

girls in families of various sizes. Their writing reflected an understanding of why the probabilities diminished as the number of children increased.

At this point, students were again asked to write what they had learned. Typical comments follow:

- I learned [that] my prediction [should] always have a meaning. I know predictions don’t have to be exactly right, but you need to think hard about all the possibilities.
- I learned that if you switch the order of happenings, it counts as a different way. I also learned that probability can be fun.

In the Community, you get two pets. The Elders pick the pets for each family. There were six choices of pets to have: dog, cat, fish, snake, bird, and hamster. What was the probability of getting a dog and a cat?

Mark

The elders decided [that] all families can have five children. How often will there be no boys?

Charlie

The Elders wanted hair color to come out even. So out of 40 people they wanted 10 to have blond hair, 10 red, 10 black, and 10 brown. Obviously, the Elders cannot just make someone have a certain color hair. What are the chances it would come out "just right"?

Elizabeth

Fig. 5 Student-generated problems

- I learned that questions you have can often be answered by doing a simple experiment.
- We found out that a larger sample usually gives you results in the experiment that are nearer the theoretical ones.

Finally, the foregoing problem related to *The Giver* was presented as a culminating assessment activity. Students were successful and justifiably proud of their growth. They discussed the real-world chances of a girl or boy being born, but all classes were satisfied that the 50-50 model was sufficient. One class insisted on designing a spreadsheet to produce row 50 of Pascal's triangle and cheered when it validated the results of their experiments. But the clearest evidence of student understanding was the content of the original problems that the students designed. Students presented them for their classmates to solve. A few samples are shown in **figure 5.**

And, finally, we present Graeme. In his seventh-grade class, Graeme was a basketball star who was reluctant to admit that he liked mathematics and even more reluctant to take risks in mathematics class. Not this time. Graeme devised the following problem:

> In the Community, people were randomly assigned a truck, jeep, or a car. There are plenty of each kind of vehicle. The people have no control over which kind they get. Each person gets one vehicle. If there are 50 people, how many will get a car?

(See **fig. 6**). He not only insisted on presenting his problem in class but also took time at lunch to make a transparency for his presentation. As

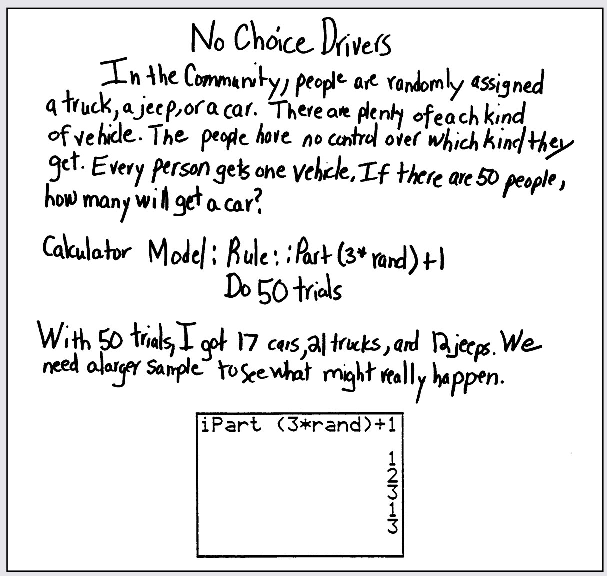


Fig. 6 Graeme's problem

Graeme proudly showed his creation to the class, I simply thought, "Slam dunk!"

Clearly, blending connections to literature with intriguing yet accessible problems and opportunities for cooperative learning and writing can increase both the interest and confidence of middle school students in dealing with probability.

References

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