Multiple Imputation

Hsueh-Sheng Wu
CFDR Workshop Series
June 13, 2022

BGSU
Center for Family and Demographic Research

Outline

- Importance of analyzing missing data
- Three mechanisms underlying missing data
- Strategies of handling missing data
- Obtain estimates from imputed data
- What is multiple imputation?
- Decisions on multiple imputation:
  - Do I really need to do multiple imputation (MI)?
  - What format of data do I want to work with?
  - What variables should be used in multiple imputation?
  - How many imputed data sets need to be created?
  - What imputation models should I use?
- Steps of imputing and analyzing missing data
- Stata example
- Conclusions
- Additional information

Importance of Analyzing Missing Data

- Missing data pose several dilemma in analyzing data:
  - If you use the original data and exclude people with missing data from the analysis, you do not use all information contained in the data.
  - If you replace missing values with other values and run the analysis, you can use all information in the data.
  - When the results differ for these two data sets, which result should you trust?

- Failure to adequately analyze missing data results in:
  - insufficient statistical power
  - upward or downward biases in parameter estimates
  - under- or over-estimated standard errors of the parameters
  - inaccurate findings

Three Mechanisms Underlying Missing Data

Assuming that we have a data set that contains one Y variable and many X variables:

- Missing completely at random (MCAR): No X variables in the data sets can predict whether the values in a variable (e.g., Y) will be missing. Also, the variable, Y, has missing value not because of the unobserved value of Y itself.
- Missing at random (MAR): X variables in the data sets can predict whether the values in Y will be missing.
- Missing not at random (MNAR): If the value of the variable, Y, or variables other than Xs determines whether the value of Y will be missing

Strategies of Handling Missing Data

- Delete cases
  - Pairwise deletion
  - Listwise deletion
- Substitution and imputation
  - Mean substitution
  - Regression
  - Multiple imputation

BGSU

Center for Family and Demographic Research

Obtain Estimates from Imputed Data

- Mean of the estimate obtained from m imputed data sets

$$\bar{Q} = \frac{1}{m}\sum_{j=1}^{m}\hat{Q}_j$$

- Standard error of the estimate obtained from m imputed data sets
  - Mean of within-imputation variance

$$\bar{U} = \frac{1}{m}\sum_{j=1}^{m}U_j$$

BGSU

Center for Family and Demographic Research

Obtain Estimates from Imputed Data (Cont.)

- Between-imputation variance

$$B = \frac{1}{m-1}\sum_{j=1}^{m}(\hat{Q}_j - \bar{Q})^2$$

- Total variance

$$T = \bar{U} + \left(1 + \frac{1}{m}\right)B$$

- Standard error of the estimate

$$\sqrt{T}$$

What Is Multiple Imputation (MI)?

- The goal of MI is to obtain the accurate parameter estimates for relations of interest.

- The missing data are imputed *m* times to create m multiple data files.

- Analysis is conducted on each of *m* imputed data sets.

- The mean and standard error of the parameter from each imputed data set are combined to obtain the final estimate of the parameter.

Do I Really Need to Do Multiple Imputation?

- You will need to do multiple imputation if many respondents will be excluded from the analytic sample due to their missing values and if the missing values of one variable can be predicted by other variables in the data file (i.e., meeting the missing on random (MAR) assumption)

- Look at the pattern of missingness in the data using Stata commands as follows:

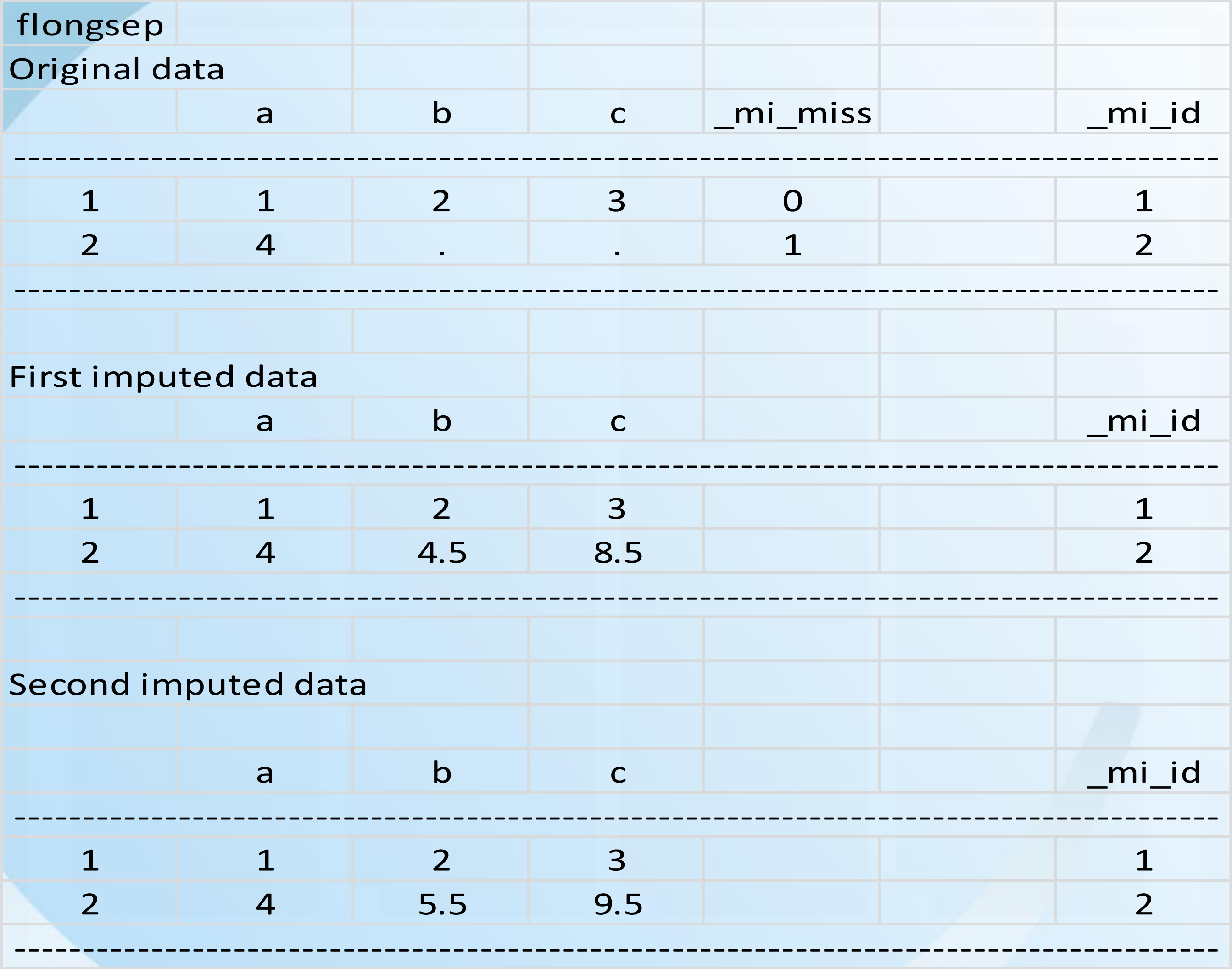
```
misstable pattern
misstable sum, all
misstable nested
```

What Format of Data Do I Want to Work With?

Four types of data format: flong, flongsep, mlong, wid

Original data						
	a	b	c			
--						
1	1	2	3			
2	4	.	.			
--						
flong						
	a	b	c	_mi_miss	_mi_m	_mi_id
--						
1	1	2	3	0	0	1
2	4	.	.	1	0	2
--						
3	1	2	3	.	1	1
4	4	4.5	8.5	.	1	2
--						
5	1	2	3	.	2	1
6	4	5.5	9.5	.	2	2

What Format of Data Do I Want to Work With? (Cont.)



flongsep						
Original data						
	a	b	c	_mi_miss		_mi_id
--						
1	1	2	3	0		1
2	4	.	.	1		2
--						
First imputed data						
	a	b	c			_mi_id
--						
1	1	2	3			1
2	4	4.5	8.5			2
--						
Second imputed data						
	a	b	c			_mi_id
--						
1	1	2	3			1
2	4	5.5	9.5			2
--						

What Format of Data Do I Want to Work With? (Cont.)

mlong								
	a	b	c	_mi_miss	_mi_m	_mi_id		
--								
1	1	2	3	0	0	1		
2	4	.	.	1	0	2		
3	4	4.5	8.5	.	1	2		
4	4	5.5	9.5	.	2	2		
--								
wide								
	a	b	c	_mi_miss	_1_b	_1_c	_2_b	_2_c
--								
1	1	2	3	0	2	3	2	3
2	4	.	.	1	4.5	8.5	5.5	9.5
--								

What Format of Data Do I Want to Work With? (Cont.)

Use “mi convert” to change between formats within Stata:

```
use style_flong.dta, clear

mi convert flongsep example, clear
list

use _1_example, clear
list

use _2_example, clear
list

mi convert mlong, clear

mi convert wide, clear
```

What Variables Should Be Used in Multiple Imputation?

- Imputation model should definitely include dependent variables, independent variables, and some auxiliary variables (i.e., interaction terms or squared terms of independent variables, and weight variables), and maybe some other auxiliary variables (i.e., variables not in your analytic models).

- These auxiliary variables might help with the imputations as they make MAR assumption more reasonable (Collins et al., 2003). Using auxiliary variables is easy when MICE, but not MVN, is used.

- If you analyze a scale score, you should impute scale items and then generate the scale score unless (1) over half of the individual scale items are observed, (2) items have high value of internal consistency, and (3) the item-total correlations are similar across items (Graham, 2008).

- Stata can impute data and take into account the weighting issues at the same time.

How Many Data Sets Need to Be Created?

- There is not consensus on this question.

- Conventional advice has been that 5 to10 imputed data sets are sufficient to impute the point estimate of missing data, and more (e.g., 40) may yield increased power in the imputation (Graham, Olchowski, & Gilreath, 2007)

- However, when it comes to estimate the standard errors of parameters, Stata manual (p.79) suggests that hundreds of imputed data sets provide reliable estimate of standard errors of parameters

What Imputation Model Should I Use?

We focus on two most general imputation models in Stata
(1) Multiple imputation with the multivariate normal model (MVN)
(2) Multiple Imputation by chained equations (MICE)

MVN:

- Assume a joint multivariate normal distribution of all variables
- Directly maximize the parameter estimate using the observed cases and maximum likelihood method
- Sometimes multivariate normal model is used even with categorical variables, but this can be severely biased (Horton, Lipsitz, and Parzen, 2003; Allison 2005).
- Can’t easily handle complexities such as skip patterns, bounds restrictions, complex designs

MICE

- Fit model of each variable, conditional on all others
- Models used depend on types of variables (categorical/continuous/binary). Researchers have more flexibility in specifying imputation models for different variables and for different subpopulations.
- Doesn’t necessarily imply a proper joint distribution like MVN does, but this doesn’t seem to be a big problem in practice

Steps of Imputing and Analyzing Missing Data

- Read in the raw data
- Examine the missingness of the data
- Specifying the model for multiple imputation
- Impute the data
- Save the imputed data
- Conduct post-estimation test

Stata Example

See the handouts

Conclusions

- Multiple Imputation helps keep as many observations as possible in the analytic analysis.

- Theoretically, if multiple imputation models are specified correctly, researchers should be able to get unbiased parameter estimates when analyzing imputed data.

- When checking the imputation model, please check for the accuracy of you codes, the functional form between the predictors and the imputed variable, and if such functional form should differ for different sub-populations.

- Doing multiple imputation can be time-consuming if you have big data files, many variables, lots of categorical variables, and complex functional forms between variables. Thus, you should start small by doing multiple imputation for a small number of variables and then expand

- If you have problems doing multiple imputation, please send an email to me (wuh@bgsu.edu) or drop by my office. I am glad to help.

Additional Information

- Azur, M; Stuart, E.; Frangakis, C.; Leaf, P (2011) Multiple Imputation by Chained Equation: What is it and how does it work? *International Journal of Methods in Psychiatric Research*, 20,40-49.

- Royston, P & White, I. (2011) Multiple Imputation by Chained Equations (MICE): Implementation in Stata. Journal of Statistical Software, 45,1-20.

- The Manual of Stata Multiple Imputation

- Youtube Videos:
  Recent Advances in missing Data Methods: Imputation and Weighting - Elizabeth Stuart (https://www.youtube.com/watch?v=xnQ17bbSeEk)
  Multiple imputation in Stata®: Setup, imputation, estimation--regression imputation (https://www.youtube.com/watch?v=i6SOlq0mjuc&index=1&list=PLN5IskQdgXWmhjxC5eopeRJwpI9G7Kp5w)
  Multiple imputation in Stata®: Setup, imputation, estimation--predictive mean matching (https://www.youtube.com/watch?v=c75E2LBGoBQ&index=2&list=PLN5IskQdgXWmhjxC5eopeRJwpI9G7Kp5w)
  Multiple imputation in Stata®: Setup, imputation, estimation--logistic regression (https://www.youtube.com/watch?v=QVvTpPx2LyU&index=3&list=PLN5IskQdgXWmhjxC5eopeRJwpI9G7Kp5w)
  Multiple Imputation in Stata: (http://www.ssc.wisc.edu/sscc/pubs/stata_mi_intro.htm)
  Recent Advances in Missing Data Methods: Multiple Imputation by Chained Equations (http://www.academyhealth.org/files/2010/sunday/StuartE.pdf)